

LINEAR GRAPH THEORETIC MODELLING STUDIES OF
LARGE-SCALE POWER SYSTEMS

by

CHANDRIKA PRASAD GUPTA

Thesis Submitted

to

The Indian Institute of Technology, Delhi

for the award of the degree of

DOCTOR OF PHILOSOPHY

Department of Electrical Engineering
Indian Institute of Technology, Delhi

January, 1975

(ii)

C E R T I F I C A T E

Certified that the research work "Linear Graph Theoretic Modelling Studies of Large-Scale Power Systems" by C.P.Gupta, has been carried out under my supervision at the Indian Institute of Technology, Delhi and that this work has not been submitted elsewhere for the award of a degree or diploma.

Dated 27/1/75

P.S. Satsangi

(P.S.Satsangi)

Professor

Department of Electrical Engineering
Indian Institute of Technology, Delhi
New Delhi-110029

(iii)

ACKNOWLEDGMENT

The author wishes to record his greatest indebtedness to Prof. Prem.S.Satsangi, his supervisor, for the able guidance and moral support readily provided through out his Ph.D. programme.

Sincere thanks are to Prof.A.K.Mahalanabis, Prof.J.Nanda, Dr.F.C.P.Bhatt, Dr.S.C.Tripathi for their constant encouragement and assistance. The author would like to gratefully acknowledge the generous help extended by Dr.Jagdish Lal, the Principal, Moti Lal Nehru Regional Engineering College,Allahabad for sanctioning study leave and authorities of I.I.T.Delhi for granting the research scholarship during the entire period of research. Sincere thanks are to Prof.M.K.Khanijo for his constant help and encouragement.

Sincere appreciation is expressed to Mr.J.N.Saini for excellent typing of the difficult manuscript and Mr.A.U.Quazi for his valuable help in drafting graphs and figures.

Thanks are due, for their constant well-wishes and fortitude, to my family and friends, in general and my wife Shyam Vati Gupta, Mrs. and Mr. R.R.Shivahare in particular, whose care and patience during the entire research schedule deserve special admiration.

-C.P.Gupta

A B S T R A C T

The linear graph theoretic power system modelling and simulation studies envisaged in this thesis are intended primarily to achieve the broad objective of developing a highly systematic and viable component-to-system construct for power system technical studies in complex frequency as well as time domain. This objective is realized through the formulation of highly generalized systems technical models which can imbed a variety of considerations of stability studies, fault studies and system operation studies into digital computations of a single integrated dynamical model which itself is amenable to computerized formulation. The organization of the research presentation in this thesis is in terms of six chapters which are briefly as follows.

In Chapter I on Introduction a background of power system planning and technical studies has been presented. In particular the literature on power system model formulation has been surveyed. Also presented are highlights of the linear graph theoretic approach for modelling large-scale systems such as power system. Finally, the chapter briefly outlines the contents of the rest of the thesis.

Chapter II develops a linearized system model in the complex frequency domain for a typical electric energy system through the linear graph theoretic technique of branch formulation employing sub-system-to-system construction philosophy of

large-scale physical systems modelling framework. In the process, linearized models of basic components such as generators, transformers, transmission lines and loads have been formulated and combined with appropriate interconnection constraint equations to develop multi-terminal subassembly representations at various levels. In order to extend the scope of such systems studies further so as to include more realistic and sophisticated situations involving the effects of governing mechanism and automatic voltage regulator, we have also developed a complex frequency domain two port model each for the sub-systems consisting of the governing mechanism of the turbine and voltage regulator.

Chapter III is devoted to formalizing a highly rigorous and systematic procedure for the formulation of a linearized state-space model for a large-scale power system with the desirable property of including only directly measurable terminal variables as state variables. The concept of multi-terminal subassembly representation furnishes an orderly and repetitive subsystem-to-system modelling format for stage-by-stage combination of the linearized state models of individual basic components of power system through their relevant interconnection constraint structure. Certain empirical stability and simulation studies have been performed in the context of a typical power system under consideration.

Chapter IV addresses itself to the formulation of a highly general nonlinear state model for a power system on a single

phase basis, using once again the concept of multi-terminal subassembly representation and the philosophy of component-to-system construction characterizing the linear graph theoretic physical system modelling framework underlying the research in this thesis. Such a nonlinear system model is eminently suited for investigating severe n -phase faults representing either dead short-circuits or short circuits through impedance. A simulation study for a 1-phase dead short circuit at alternator terminals has been empirically implemented under the assumption of a constant applied torque at the shaft and a constant applied voltage across the field terminals.

Chapter V presents the most general nonlinear state model formulation for a true 3-phase representation of the synchronous generator (with neutral grounded through an impedance) as a 6-port device. This model can be utilized for studying symmetrical and asymmetrical (line-to-ground, line-to-line, and double line-to-ground) faults with or without impedance. Since this model is exact in nature, it is also ideally suited for rigorous stability analyses. For the purpose of illustrating the applicability of the model to fault situations a single-line-to-ground fault and a 3ϕ balanced fault on an unloaded alternator have been studied.

Finally, in Chapter VI, the main contributions of the thesis in the area of modelling and simulation studies of large scale power systems are discussed together with a brief presentation of the scope of further work in the field uncovered by the present research.

(vii)

Curriculum Vitae

Name: ... Chandrika Prasad Gupta

Place & Date of Birth: January 4, 1932 - Baberu, Uttar Pradesh, India.

Marital Status: Married, 2 Children

Experience:

Academic Since 1963 Reader, Department of Electrical Engg, Motilal Nehru Regional Engineering College, Allahabad (UP).

1958-1963 Lecturer, Department of Electrical Engg, Banaras Hindu University, Varanasi (UP).

1957-1958 Lecturer in Electrical Engineering, Mining Training School, Kurasia Colliery (MP)

Professional:

1956-1957 Electrical Supervisor Hirakud Dam Project Orissa.

Training: Feb. 1962 to Feb. 1963 Advanced postgraduate training in the manufacturing of H.T. equipments at BUDAPEST (Hungary).

Education B.E.(Elec) University of Rajasthan 1956
M.E.(Elec Power) Indian Institute of Science, Bangalore 1962

Membership of

Professional

Societies: Student Member: Institute of Electrical and Electronics Engineers, U.S.A.

Member of Indian Society of Technical Education.

TABLE OF CONTENTS

| | | | |
|-------------------|---|-----|--------|
| CERTIFICATE | ... | ... | (ii) |
| ACKNOWLEDGMENT | ... | ... | (iii) |
| ABSTRACT | ... | ... | (iv) |
| CURRICULUM VITAE | ... | ... | (vii) |
| TABLE OF CONTENTS | ... | ... | (viii) |
| <u>CHAPTER-I</u> | INTRODUCTION | ... | 1 |
| 1.1 | Background | ... | 1 |
| 1.2 | Large-scale Systems Viewpoint of a Power Systems. | ... | 6 |
| 1.3 | Survey of Existing Models. | ... | 8 |
| | 1.3.1 Historical perspective of power system models. | ... | 8 |
| | 1.3.2 Synchronous machine representation | ... | 11 |
| | 1.3.3 Recent multimachine power system models. | ... | 15 |
| 1.4 | Linear Graph Based Physical System Theory .. | ... | 17 |
| | 1.4.1 Lineargraph theory in systems perspective. | ... | 17 |
| | 1.4.2 Systems concept | ... | 19 |
| | 1.4.3 Symbolic branch, chord and branch-chord formulation | ... | 24 |
| | 1.4.4 State-space formulation | ... | 30 |
| | 1.4.5 Distinguishing features | ... | 39 |
| 1.5 | Scope of Thesis | ... | 42 |
| <u>CHAPTER II</u> | COMPLEX FREQUENCY DOMAIN MODELLING FRAMEWORK FOR VARIATIONAL ANALYSIS | | |
| 2.1 | Introduction | ... | 46 |
| 2.2 | A Typical Power System | ... | 49 |
| 2.3 | Sub-sub-system to Sub-system Construct | ... | 51 |

| | | | | |
|---|--|-----|-----|-----|
| 2.4 | Component Models | ... | ... | 53 |
| 2.4.1 | Generator representation | | ... | 53 |
| 2.4.2 | Transformer representation | | ... | 58 |
| 2.4.3 | High energy transmission line representation | ... | ... | 59 |
| 2.4.4 | Load representation | | ... | 60 |
| 2.5 | Subassembly Model | ... | ... | 61 |
| 2.5.1 | Model of G_1 and Tr Sub-Sub-System | | ... | 62 |
| 2.5.2 | (a) Sub-system representation at the i th level | ... | ... | 66 |
| | (b) Subsystem representation at the $(i+1)$ th level | ... | ... | 67 |
| 2.5.3 | Model of G_1, T_r, G_2 and Tl_1 subassembly | | | 72 |
| 2.5.4 | Model of G_1, T_r, G_2, Tl_1 and G_3 subassembly. | | | 72 |
| 2.5.5 | Model of G_1, T_r, G_2, Tl_1, G_3 and Tl_2 sub-assembly | ... | ... | 73 |
| 2.5.6 | Final system model excluding load. | | ... | 74 |
| 2.6 | Typical Power System Variational Model. Eight-port Representation. | ... | ... | 75 |
| 2.7 | Final Six-Port Representation From 8-Port Representation. | ... | ... | 82 |
| 2.8 | Concluding Remarks on Variational System Analysis | ... | ... | 83 |
| 2.9 | Power System Control Problem. | | ... | 85 |
| 2.9.1 | An S-domain 2 port model of governing mechanism of steam turbine | | ... | 87 |
| 2.9.2 | An S-domain 2-port model of the automatic voltage regulator | | ... | 94 |
| 2.10 | Concluding note | ... | ... | 101 |
| <u>CHAPTER-III</u> LINEARIZED STATE SPACE MODEL WITH APPLICATIONS | | | | |
| 3.1 | Introduction | ... | ... | 102 |
| 3.2 | State-Space Representation of Component Model | ... | ... | 107 |

| | | | |
|-------|---|-----|-----|
| 3.2.1 | Synchronous generator | ... | 107 |
| 3.2.2 | Transformer | ... | 110 |
| 3.2.3 | High energy transmission line | ... | 111 |
| 3.2.4 | Load representation(static) | ... | 113 |
| 3.3 | The Subassembly Model | ... | 114 |
| 3.3.1 | State model of G_1 and T_r subassembly | ... | 114 |
| 3.3.2 | Model of G_1, T_r and G_2 subassembly | ... | 118 |
| 3.3.3 | Model of G_1, T_r, G_2 and T_{l1} , subassembly... | ... | 121 |
| 3.3.4 | Model of G_1, T_r, G_2, T_{l1} and G_3 sub-assembly | ... | 123 |
| 3.4 | Degenerate case | ... | 125 |
| 3.4.1 | Model of $G_1, T_r, G_2, T_{l1}, G_3$ and T_{l2} sub-assembly | ... | 125 |
| 3.4.2 | Model of overall system excluding load. | ... | 130 |
| 3.5 | State-model of the Typical Power System | ... | 135 |
| 3.6 | Stability studies Example | ... | 137 |
| | i) Single machine case | ... | 137 |
| | ii) Twomachine problem. | ... | 146 |
| 3.7 | Onemachine System Connected to Infinite Busbar through a Short Transmission Line. | ... | 156 |
| 3.7.1 | Formulation for an Infinite bus | ... | 157 |
| 3.8 | Conclusion | ... | 159 |

CHAPTER-IV NONLINEAR STATE MODEL DESCRIPTION FOR SINGLE PHASE FAULT STUDIES

| | | | |
|-------|-----------------------------------|-----|-----|
| 4.1 | Introduction | ... | 161 |
| 4.2 | Component Model Representation | ... | 163 |
| 4.2.1 | Synchronous generator | ... | 163 |
| 4.2.2 | Transformer | ... | 166 |
| 4.2.3 | Transmission line | ... | 166 |
| 4.2.4 | Load (static) | ... | 166 |
| | Case I Resistive Inductive load | ... | 166 |
| | Case II Resistive capacitive load | ... | 167 |

| | | | | |
|-------|--|-----|-----|-----|
| 4.3 | The Sub-assembly Model | ... | ... | 167 |
| 4.3.1 | Nonlinear state model of G_1 and T_r subassembly. | ... | ... | 167 |
| 4.3.2 | Model of G_1, T_r and G_2 subassembly | ... | ... | 169 |
| 4.3.3 | Model of $G_1, T_r, G_2, T_{\ell 1}$ and G_3 sub-assembly. | ... | ... | 172 |
| 4.4 | Degenerate cases | ... | ... | 174 |
| 4.4.1 | Model of $G_1, T_r, G_2, T_{\ell 1}, G_3$ and $T_{\ell 2}$ subassembly | ... | ... | 174 |
| 4.4.2 | Model of the overall typical power system without load. | ... | ... | 177 |
| 4.5 | Nonlinear state model of the typical Power System including loads. | ... | ... | 181 |
| 4.6 | Single-phase Fault Study and Empirical Illustration. | ... | ... | 185 |
| 4.6.1 | Formulation for short circuit studies (fault through impedance) | ... | ... | 187 |
| 4.6.2 | Post fault-clearance operation study in a single phase machine. | ... | ... | 193 |
| 4.7 | Conclusion | ... | ... | 194 |

CHAPTER-V NONLINEAR STATE MODEL DESCRIPTION OF A SYNCHRONOUS GENERATOR FOR SYMMETRICAL AND ASYMMETRICAL FAULT STUDIES.

| | | | | |
|-------|--|-----|-----|-----|
| 5.1 | Introduction | ... | ... | 196 |
| 5.2 | Mathematical System Modelling | ... | ... | 198 |
| 5.2.1 | Single machine system configuration | ... | ... | 198 |
| 5.2.2 | Mathematical model | ... | ... | 198 |
| 5.2.3 | State model of grounding impedance Z_7 | ... | ... | 206 |
| 5.3 | Model of Alternator With Grounded Neutral | ... | ... | 207 |
| 5.4 | Asymmetrical Fault Study and Empirical Illustration. | ... | ... | 209 |

| | | | |
|--------------------------------------|--|-----|------|
| 5.4.1 | Formulation for a single line-to-ground short circuit study(fault through impedance) | ... | 210 |
| 5.4.2 | 3-Phase fault through impedance | ... | 214 |
| 5.5 | Conclusion | ... | 216 |
| <u>CHAPTER-VI</u> CONCLUSION | | | |
| 6.1 | Contribution | ... | 218 |
| 6.2 | Scope for Further Work | ... | 222 |
| | REFERENCES | ... | R1 |
| APPENDICES AND COMPUTER PROGRAMS | | | |
| Appendix ^A -A | Linear Graph Theory: Relevant Definitions. | ... | A.1 |
| Appendix-B | Derivation of complex frequency domain Component Models. | | |
| B.1 | Synchronous generator. | ... | A.3 |
| B.2 | Transformer. | ... | A.8 |
| B.3 | Transmission lines | ... | A.9 |
| B.4 | Load representation(static) | ... | A.11 |
| Appendix-C | Derivation of linearized time domain (state-space) Component models. | | |
| C.1 | Synchronous generator | ... | A.1 |
| C.2 | Transformer | ... | A.1 |
| C.3 | Transmission line | ... | A.1 |
| C.4 | Load representation (static) | ... | A.2 |
| Appendix-D | Computer Programmes | | |
| D.1 | Polyadd subroutine | ... | A.2 |
| D.2 | Forest generator programme | ... | A.2 |
| D.3 | Eigenvalue calculation programme | ... | A.3 |

| | | | |
|------------|---|-----|------|
| D.4 | Simulation of Linear State Equations by Rungekutta. | ... | A.31 |
| D.5 | Simulation of Nonlinear State Equations by 4th Order Runge-Kutta. | ... | A.31 |
| Appendix-E | Partitioning Technique for Matrix Inversion. | ... | A.31 |
| Appendix-F | Chord Formulation of Multi-terminal Representation. | ... | A.31 |

