DEVELOPMENT OF COMPUTATIONALLY EFFICIENT TECHNIQUES FOR INSTANTANEOUS AND TIME-DEPENDENT ANALYSIS OF REINFORCED CONCRETE BEAMS AND FRAMES AT SERVICE LOAD

PATEL KASHYAPKUMAR ARVINDBHAI

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI

JULY 2016

DEVELOPMENT OF COMPUTATIONALLY EFFICIENT TECHNIQUES FOR INSTANTANEOUS AND TIME-DEPENDENT ANALYSIS OF REINFORCED CONCRETE BEAMS AND FRAMES AT SERVICE LOAD

by

PATEL KASHYAPKUMAR ARVINDBHAI
Department of Civil Engineering

Submitted
in fulfillment of the requirements for the degree of Doctor of Philosophy

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

JULY 2016

Dedicated to
My Parents

CERTIFICATE

This is to certify that the thesis entitled, **"Development of Computationally Efficient Techniques for Instantaneous and Time-dependent Analysis of Reinforced Concrete Beams and Frames at Service Load"** being submitted by **Mr. Patel Kashyapkumar Arvindbhai** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy** is a bonafide record of research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirement for the degree of **Doctor of Philosophy**.

The results contained in this thesis have not been submitted, in part or full, to any other University or Institute for the award of any degree or diploma.

A. K. Nagpal	**Sandeep Chaudhary**
Emeritus Professor	Associate Professor
Department of Civil Engineering	Department of Civil Engineering
Indian Institute of Technology Delhi	Malaviya National Institute of Technology
Hauz Khas, New Delhi 110016, India	JLN Marg, Jaipur 302017, India

ACKNOWLEDGEMENTS

I express my deep sense of gratitude towards my supervisor Prof. A. K. Nagpal for giving me an opportunity to carry out my research work under his supervision. He inspired and motivated me for the present work at every stage with invaluable suggestions. I have found him ready to help solving my smallest doubts at any moment despite his extremely busy schedule. No amount of appreciation can be good enough to express my gratitude and indebtedness to him.

I am very much thankful to my co-supervisor, Dr. Sandeep Chaudhary, for inspiration and motivation to join the research programme and for his excellent guidance in the research work. His patience in clarifying my doubts and the perfectionism he maintains in the work, have immensely helped me in the research. His family has been very caring and supportive towards me. I always felt like being at home during my stay with them at Jaipur.

Besides my supervisors, I am grateful to Prof. Shashi Mathur, Prof. B. Bhattacharjee, Prof. D. K. Sehgal, Prof. T. K. Datta, Prof N. K. Garg, Prof. G. V. Ramana, Dr. K. N. Jha, and Dr. Vasant Matasagar for their support, motivation, and encouragement.

I take this opportunity to express my thanks to Dr. M. P. Ramnavas, Dr. K. K. Jain, Dr. Kaustav Sarkar, Mr. Hemant Shrivastava, Mr. Ankit Bhardwaj, and Mr. Amit Kumar, without their continuous support and moral boosting the journey would have been much difficult. Special thanks are due to my well wishers, Pankaj, Pratik, Rahul, Gaurav, Sushil, Rohit, and Vaishakh, for their cooperation and support. I am thankful to Mr. N. R. Gehlot, Mr. Amit Bundela, Mr. Rajveer Aggarwal, Mr. Bikram Chand, and Mr. Randhir Jha for providing me the necessary support in the computational laboratory and departmental office.

I have a galaxy of friends who have been the source of my continuous strength during the course of this work some of the outstanding names amongst those are: Devang, Chirag, Parth, Mehul, Suchinder, Love, Vijay, Jago, Manoj, Manish S, Shailesh, Jignesh, Manish G, and Swetang. I express my thanks towards all my friends who made this arduous task a pleasurable experience indeed.

I would like to acknowledge the blessings of my parents with thanks for all they have done for me. They have sacrificed a lot for me throughout my life. I am also thankful to my family members, especially my elder brother cum colleague Dr. Dilip A. Patel for inspiration, motivation, and strength in the tough time during the programme. Finally, I cannot forget to mention the encouragement, patience, love given by my beloved wife, Nikita who gave me the strength at a stretch during the course of this work.

Patel Kashyapkumar Arvindbhai

ABSTRACT

Reinforced Concrete (RC) members are widely used in the construction of buildings and bridges. At service load, concrete in tensile zone generally cracks owing to moment in the zone being higher than the cracking moment. This cracking may result in change in deflection, considerable moment redistribution along the member length, and stress redistribution across the cross-sections. Further, under sustained load, the time-effects (creep and shrinkage) in the concrete may lead to progressive cracking, increase in deflection, moment and stress redistribution. The appropriate prediction of instantaneous as well as time-dependent behaviour of RC beams and frames at service load, considering the concrete cracking is therefore important. Of the two approaches available in the literature for the analysis at service load stage, numerical approach requires huge computational effort whereas empirical approach may not be accurate and also does not take into account all the aspects. For application to large RC structures, development of computationally efficient techniques that require a minimal computational effort but give accuracy that is acceptable for practical applications is desirable.

In the present study, first, a computationally efficient analytical-numerical procedure has been developed to take into account the non-linear effects of concrete cracking and time-dependent effects of creep and shrinkage in RC beams and frames subjected to service load. A typical beam/frame member is modeled as a single element and is visualized to consist of at the most five zones (cracked or uncracked). The proposed element has been used to develop the procedure. The closed form expressions for flexibility and stiffness coefficients, end displacements, load vector, cracked lengths, and mid-span deflection of the element have been derived. The procedure is

analytical at the element level and numerical at the structural level. The procedure yields inelastic deflections, redistributed moments as well as inelastic cracked lengths and corresponding interpolation coefficients for tension stiffening. The proposed procedure has been validated with experimental and other results available in the literature and with finite element method (FEM) results. The procedure leads to a considerable saving in computational time in case of large RC structures e.g. tall RC building frames. The procedure, at minimal computational effort and reinforcement input data, yields the results that are close to experimental and FEM results. Using the proposed procedure, the effect of variations in tensile and compressive reinforcements on the instantaneous and time-dependent behaviour of RC beams and frames has been studied.

Next, a methodology has been presented to develop the neural networks for rapid prediction of the inelastic bending moments and deflections in RC beams and frames. In the present study, neural network models have been developed to predict the instantaneous inelastic deflections and bending moments in RC beams considering concrete cracking and tension stiffening. For development of these neural networks, tens of thousands of analysis are required to generate data sets. The computational efficiency of the proposed procedure is utilised to carry out such large number of analyses. Closed form expressions are obtained from the weights and biases of the developed neural networks. The proposed expressions predict the inelastic moments and deflections (incorporating concrete cracking and tension stiffening) from the elastic moments and deflections (neglecting concrete cracking and tension stiffening) in RC beams. The elastic moments and deflections can be obtained from any of the readily available software. The inelastic deflections and moments obtained from the proposed expressions are compared with those obtained from the FEM for example beams of

different number of spans and cross-section properties and the errors are found to be small.

Further, a sophisticated two stage procedure, which uses both the proposed methodology of using neural networks and the analytical-numerical procedure, has been proposed for instantaneous analysis of beams considering concrete cracking and tension stiffening. The analytical-numerical procedure requires around six analyses to yield results with sufficient accuracy for design purpose. In the two stage procedure, only two analyses, one in each stage, are required to yield results with similar accuracy.

LIST OF CONTENTS

Chapter 1

Chapter 2

ANALYTICAL-NUMERICAL PROCEDURE INCORPORATING CRACKING IN REINFORCED CONCRETE BEAMS...……………………... 46

Chapter 3

ANALYTICAL-NUMERICAL PROCEDURE INCORPORATING CRACKING AND TIME-EFFECTS IN REINFORCED CONCRETE BEAMS…………………………………………………...……………………... 86

Chapter 4

ANALYTICAL-NUMERICAL PROCEDURE INCORPORATING CRACKING AND TIME-EFFECTS IN REINFORCED CONCRETE FRAMES……………………………………………………………… 161

Chapter 5

RAPID PREDICTION OF INELASTIC DESIGN QUANTITIES IN REINFORCED CONCRETE BEAMS CONSIDERING CRACKING……….... 220

Chapter 6

TWO STAGE PROCEDURE FOR INSTANTANEOUS ANALYSIS OF REINFORCED CONCRETE BEAMS CONSIDERING CRACKING…...….. 260

Chapter 7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK…… 299

Appendix A

Appendix B

Appendix C

Appendix D

Appendix E

LIST OF FIGURES

LIST OF TABLES

LIST OF NOTATIONS

Alphabetic characters

A, A_c, A_e : areas;

A_{st}, A_{sb} : area of top and bottom steel reinforcement respectively;

A_v : shearing steel area;

A_{st}^i, A_{sb}^i : area of top and bottom reinforcement respectively, in a layer (i^{th}) in a region;

B, B_c, B_e : first moment of areas about reference axis;

B_f, B_w : width of flange and web respectively;

C : inertia ratio;

D : total depth of section;

D^{el}, D^{in} : elastic and inelastic deflection respectively;

D^{ss} : elastic deflection of an equivalent simply supported beam;

E : modulus of elasticity;

G : shear modulus of concrete;

I, I_c, I_e : second moment of areas about reference axis;

I_j : j^{th} input parameter;

I^{un}, I^{cr} : moment of inertia uncracked and cracked section about neutral axis respectively;

K : shearing rigidity;

L : span length;

L_{cr} : length of the cracked zone;

M_{cr} : cracking moment;

M^{el}, M^{in} : elastic and inelastic moment respectively;

$M^{it}, M_A^{it}, M_B^{it}, M^{it}(t_j),$ $M^t, M^t(t), M^t(t_j)$: bending moments;

$\Delta M, \Delta M^{c}, \Delta M^{id}, \Delta M_A^{id}, \Delta M_B^{id}, \Delta M^{s}, \Delta M_A^{id}(t_j), \Delta M_B^{id}(t_j), \Delta M^{it}, \Delta M_A^{it}, \Delta M_B^{it}, \Delta M_A^{it}(t_j), \Delta M_B^{it}(t_j)$: changes in bending moments;

$N_A^{it}, N_B^{it}, N_A^{it}(t_j), N_B^{it}(t_j), N^{t}, N^{t}(t_j)$: axial forces;

$\Delta N, \Delta N^{c}, \Delta N^{id}, \Delta N_A^{id}, \Delta N_B^{id}, \Delta N^{s}, \Delta N_A^{id}(t_j), \Delta N_B^{id}(t_j), \Delta N^{it}, \Delta N_A^{it}, \Delta N_B^{it}, \Delta N_A^{it}(t_j), \Delta N_B^{it}(t_j)$: changes in axial forces;

O : output parameter;

O_m : output parameter for prediction the cracked length and interpolation coefficient at in-span location;

O^m : output parameter for prediction the cracked length and interpolation coefficient at support;

$\{P^0\}, \{P^0(t_j)\}$: fixed end force vectors for uncracked beam;

$\{P^{er}\}, \{P^{er}(t_j)\}$: residual force vectors;

R : cracking moment to elastic moment ratio;

$R_A^{it}, R_B^{it}, R_A^{it}(t_j), R_B^{it},(t_j)$: end reactions;

$\Delta R_A^{id}, \Delta R_B^{id}, \Delta R_A^{id}(t_j), \Delta R_B^{id}(t_j), \Delta R_A^{it}, \Delta R_B^{it}, \Delta R_A^{it}(t_j), \Delta R_B^{it}(t_j)$: changes in end reactions;

S : stiffness;

$bias$: bias of hidden or output neuron;

d : effective depth of section;

$d_m, d_m^{it}, d_m^{it}(t_j), d_m^t, d_m^t(t_j)$: mid-span deflections;

$\Delta d_m, \Delta d_m^c, \Delta d_m^c(t_j), \Delta d_m^c(t_{i+1}, t_i, t_j), \Delta d_m^{id}, \Delta d_m^{id}(t_j), \Delta d_m^{it}(t_j), \Delta d_m^s, \Delta d_m^s(t_{i+1}, t_i, t_j), \Delta d_{c,m}^c(t_{i+1}, t_i, t_j), \Delta d_{s,m}^c(t_{i+1}, t_i, t_j)$: changes in mid-span deflections;

d_t, d_b : effective concrete cover at top and bottom respectively;

$\{d^{er}\}, \{d^{er}(t_1)\}$: error or difference in displacement vectors;

$\{d^*\}, \{d^{it,*}(t_1)\}$: revised displacement vectors;

f : shear coefficient;

f_{ij} : flexibility coefficients;

f_s : stress in steel reinforcement on tension face;

f_t, $f_c^{'}$: tensile strength and cylinder compressive strength of concrete at 28 days respectively;

h_k : k^{th} hidden neuron;

k_{ij} : stiffness coefficients;

$[k], [k_e(t_i, t_j)]$: stiffness matrices;

l_t^i, l_b^i : length of top and bottom reinforcement respectively, in a layer (i^{th}) in a region;

n : modular ratio or number of spans in a beam;

n_t, n_b : number of top and bottom layers respectively, in a region;

p : half band width;

$\{p\}, \{p^{er}(t_j)\}, \{p^{it}(t_j)\}$: force vectors of an element;

q : total number of DOF or number of input parameters;

r : number of hidden neurons;

s : spacing of reinforcements;

t, t_j : time instants;

$u_A^{it}, u_A^{it,*}(t_j),$
$u_B^{it,*}(t_j), u_{A,r}^{it,*}(t_j)$: axial displacements;

$\Delta u_A^c, \Delta u_A^c(t_{i+1}, t_i, t_j),$
$\Delta u_A^s, \Delta u_A^s(t_{i+1}, t_i, t_j)$: changes in axial displacements;

w, w_{cr} : uniformly distributed load and cracking uniformly distributed load respectively;

$w_{j,k}^{ih}$: weight of the link between I_j and h_c;

$w_{k,1}^{ho}$: weight of the link between h_c and O;

x : cracked length; and

z : length of a region.

Greek letters

$\beta_{un}^{c,m}, \beta_{un}^{c,n}, \beta_{c,un}^{c,m}, \beta_{c,un}^{c,n},$
$\beta_{un}^{c,m}(t_i,t_j), \beta_{un}^{c,n}(t_i,t_j),$
$\beta_{c,un}^{c,m}(t_i,t_j), \beta_{c,un}^{c,n}(t_i,t_j)$: creep factors for curvature in a cross-section of an uncracked zone; similarly, subscript cr indicates creep factors for curvature the cracked state in a cross-section of cracked zone;

$\Delta\beta_{un}^{c,m}(t_{i+1}, t_i, t_j),$
$\Delta\beta_{un}^{c,n}(t_{i+1}, t_i, t_j),$
$\Delta\beta_{c,un}^{c,m}(t_{i+1}, t_i, t_j),$
$\Delta\beta_{c,un}^{c,n}(t_{i+1}, t_i, t_j)$: changes in creep factors for curvature in a cross-section of an uncracked zone; similarly, subscript cr indicates changes in creep factors for curvature the cracked state in a cross-section of cracked zone;

$\varepsilon, \varepsilon_0^{it}, \varepsilon_0^{it}(x),$
$\varepsilon_0^{it,m}, \varepsilon_0^{it,n}, \varepsilon_{0,un}^{it},$
$\varepsilon_{0,cr}^{it}, \varepsilon_{0,ts}^{it}, \varepsilon_{0,un}^{t},$
$\varepsilon_{0,ts}^{t}, \varepsilon_{0,un}^{it}(t_j),$
$\varepsilon_{0,cr}^{it}(t_j), \varepsilon_{0,ts}^{it}(t_j),$
$\varepsilon_{0,un}^{t}(t_j), \varepsilon_{0,ts}^{t}(t_j)$: top fiber strains;

$\Delta\varepsilon^{c}_{0,un}, \Delta\varepsilon^{c}_{0,un}(t_{i+1}, t_i, t_j),$ $\Delta\varepsilon^{id}_{0,un}, \Delta\varepsilon^{id}_{0,un}(t_j), \Delta\varepsilon^{id}_{0,cr},$ $\Delta\varepsilon^{id}_{0,cr}(t_j), \Delta\varepsilon^{s}_{0,un},$ $\Delta\varepsilon^{s}_{0,un}(t_j, t_i),$ $\Delta\varepsilon^{s}_{0,un}(t_{i+1}, t_i, t_j)$	: changes top fiber strains;
ε_t, ε_u, ε_y	: cracking strain, maximum tensile strain, and strain at a distance y from the reference axis respectively;
$\varepsilon^{sh}, \varepsilon^{sh}(t_i, t_j)$	: shrinkage strains;
$\eta, \eta_i, \eta_A, \eta_B, \eta_i(t_j),$ $\eta_A(t_j), \eta_B(t_j)$	: average interpolation coefficient;
$\theta_A, \theta_B, \theta^{it}_A, \theta^{it}_B,$ $\theta^{it,*}_A(t_j), \theta^{it,*}_B(t_j),$ $\theta^{it,*}_{A,n}(t_j), \theta^{it,*}_{B,n}(t_j)$	: end rotations;
$\Delta\theta^{c}_A(t_{i+1}, t_i, t_j),$ $\Delta\theta^{c}_B(t_{i+1}, t_i, t_j),$ $\Delta\theta^{s}_A(t_{i+1}, t_i, t_j),$ $\Delta\theta^{s}_B(t_{i+1}, t_i, t_j)$	: changes in end rotations;
κ	: coefficient representing influence of duration of application or repetition of loading on interpolation coefficient;
λ	: tolerance value;
$\lambda^{c,m}_{un}, \lambda^{c,n}_{un}, \lambda^{c,m}_{c,un},$ $\lambda^{c,n}_{c,un}, \lambda^{c,m}_{un}(t_i, t_j),$ $\lambda^{c,n}_{un}(t_i, t_j), \lambda^{c,m}_{c,un}(t_i, t_j),$ $\lambda^{c,n}_{c,un}(t_i, t_j)$	: creep factors for strain in a cross-section of an uncracked zone; similarly, subscript *cr* indicates creep factors for strain the cracked state in a cross-section of cracked zone;
$\Delta\lambda^{c,m}_{un}(t_{i+1}, t_i, t_j),$ $\Delta\lambda^{c,n}_{un}(t_{i+1}, t_i, t_j),$ $\Delta\lambda^{c,m}_{c,un}(t_{i+1}, t_i, t_j),$ $\Delta\lambda^{c,n}_{c,un}(t_{i+1}, t_i, t_j)$	: changes in creep factors for strain in a cross-section of an uncracked zone; similarly, subscript *cr* indicates changes in creep factors for strain the cracked state in a cross-section of cracked zone;

μ : Poisson’s ratio of concrete;

$\xi, \xi_i, \xi_A, \xi_B, \xi_i(t_j), \xi_A(t_j), \xi_B(t_j)$: average interpolation coefficient;

$\rho, \rho(x), \rho^{it}, \rho^{it}(x), \rho^{it,m}, \rho^{it,n}, \rho^{it}_{un}, \rho^{it}_{cr}, \rho^{it}_{ts}, \rho^{t}_{un}, \rho^{t}_{ts}, \rho^{it}_{un}(t_j), \rho^{it}_{cr}(t_j), \rho^{it}_{ts}(t_j), \rho^{t}_{un}(t_j), \rho^{t}_{ts}(t_j)$: curvatures;

$\Delta\rho^{c}_{un}, \Delta\rho^{c}_{un}(t_{i+1}, t_i, t_j), \Delta\rho^{id}_{un}, \Delta\rho^{id}_{un}(t_j), \Delta\rho^{id}_{cr}, \Delta\rho^{id}_{cr}(t_j), \Delta\rho^{s}_{un}, \Delta\rho^{s}_{un}(t_j, t_i), \Delta\rho^{s}_{un}(t_{i+1}, t_i, t_j)$: changes in curvatures;

ρ_{st}, ρ_{sb} : percentage steel reinforcement at top and bottom respectively;

$\sigma, \sigma^{it}_0, \sigma^{it}_0(x), \sigma^{it,m}_0, \sigma^{it,n}_0, \sigma^{it}_{0,un}, \sigma^{it}_{0,cr}, \sigma^{it}_{0,ts}, \sigma^{t}_{0,un}, \sigma^{t}_{0,ts}, \sigma^{it}_{0,un}(t_j), \sigma^{it}_{0,cr}(t_j), \sigma^{it}_{0,ts}(t_j), \sigma^{t}_{0,un}(t_j), \sigma^{t}_{0,ts}(t_j)$: top fiber stresses;

υ_v : shearing steel content;

$\phi, \phi(t_j, t_i)$: creep coefficients; and

χ : aging coefficient.

Subscripts

0 : top fiber of beam or left face of column;

A, B : end A and end B of an element;

D : bottom fiber of beam or right face of column;

S : in-span position of an element;

c : concrete;

cr : cracked state;
e : age-adjusted;
f : flange;
i : i^{th} span;
j : input neuron number;
k : hidden neuron number or function number;
o : output neuron number;
s : steel;
ts : tension stiffening;
un : uncracked state;
w : web; and
y : distance from top fiber of section.

Superscripts

b : bottom fiber;
c : creep;
cr : cracked state;
el : elastic;
g : gross;
ho : connection between hidden and output layers;
i : i^{th} support;
id : indeterminate;
ih : connection between input and hidden layers;
in : inelastic;
it : instantaneous;
m : moment;
n : axial force;
s : shrinkage;
t : top fiber; and
un : uncracked state.

Parentheses

one term : indicates time instant at which the quantity is evaluated or assumed to arise;

two terms : the first term indicates the time instant at which a quantity is evaluated whereas the second term indicates the time of initiation of the cause owing to which the quantity arises; and

three terms : first and second terms indicate the time of the end and the beginning of the interval and the third term indicates time of initiation of a cause from which the change arises.