

**STUDY OF TIME EVOLUTION OF
PLASMA DISTRIBUTION FUNCTION
IN PAUL TRAPS**

VARUN SAXENA



DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY DELHI

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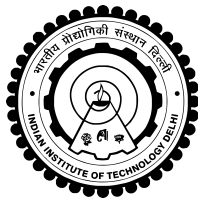
by

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DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfillment of the requirements of the degree of Doctor of philosophy



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APRIL 2018

Certificate

This is to certify that the thesis titled **Study of Time Evolution of Plasma Distribution function in Paul Traps** being submitted by **Mr. Varun Saxena** for the award of **Doctor of Philosophy** in Department of Electrical Engineering is a record of bona-fide work carried out by him under my guidance and supervision at the Department of Electrical Engineering, Indian Institute of Technology Delhi. The work presented here has not been submitted, either in part or full, for the award of any other degree or diploma

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A quote from Alfred, Lord Tennyson : Ulysses

*Tho' much is taken, much abides; and though
We are not now that strength which in old days
Moved earth and heaven; that which we are, we are;
One equal temper of heroic hearts,
Made weak by time and fate, but strong in will
To strive, to seek, to find, and not to yield.*

Dedication

This Thesis is dedicated to my father

(Late) Shri R. K. Saxena (1954-2010)

and

my mother

Smt. Neerja Saxena

Acknowledgements

Great things in life demand great love. A PhD degree is one of the few great things that have happened in my life. Starting at a time of great personal misery this degree is a proof that all things great demand perseverance & if you persevere it can reward you with unbound joy and achievement.

This acknowledgement I feel is an opportunity for me to thank all those people who have helped me achieve this landmark of academic excellence. I begin by thanking my parents who in spite of all the odds created an environment for me to grow, learn and thrive. I am especially grateful to my father who was always very encouraging of all my endeavors in life. It was his dream to see his son achieve some form of excellence in whichever field he choose for himself. I believe that this PhD degree, highest in academic education, would have surely fulfilled his dream. Even though he is no more with me, I am sure he would have been extremely happy to have seen me reach this point in my life. To my mother who has been a beacon of my hope, love, light and life. I have no words to express my gratitude. Without her all this would not have been possible. I am also grateful for the support and affection I got from my younger brother Vineet Saxena and his wife Sakshi Saxena. Mr. Sumit Khandelwal, Ms. Gargi Kaushik, Ms. Pooja Prakash and Ms. Neha Vaish and Mr. Chandra Shanker, my friends from work have always been very supportive. They always helped me in whichever way they could during my PhD. I value their presence in my life. my friend from work has always been very supportive and helped me in whatever way he could during my PhD. I value his presence in my life.

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Association with my supervisor Dr. Kushal K. Shah was perhaps the most impor-

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(Varun Saxena)

Abstract

Single particle dynamics in radio frequency traps has been extensively researched for several decades and many interesting properties and applications have been brought to the fore. However, collective dynamics of the single species charged particles inside a Paul trap is not very well understood. The understanding of the phenomena of plasma heating on account of the applied radio frequency electric field is still in its infancy. This thesis theoretically investigates the dynamics of a non-neutral plasma inside a Paul trap and provides a mathematical framework to understand the plasma temperature variations, which will hopefully aid researchers in better understanding the phenomena of radio frequency heating.

Firstly, we analyze a single species plasma in a Paul trap wherein space charge effects have been neglected and the plasma is considered to be collision-less. These are reasonable conditions to impose in the analysis of a dilute single species plasma. We begin by constructing an analytical time-dependent plasma distribution function which, for a carefully chosen initial form of q -Gaussian type Tsallis distribution and scale length, becomes time-periodic with the same frequency as that of the applied radio frequency electric field. Experimental observation of the existence of power law tails in the particle distribution is one of the primary motivation behind choosing a Tsallis distribution. The time averaged distribution function shows a double hump beyond a certain spatial threshold and the double hump moves away from the bulk as the Tsallis parameter q increases, thereby indicating that the q -Gaussian distribution is perhaps more stable as compared to the Maxwellian distribution which is obtained from Tsallis distribution when $q = 1$. Also, the plasma temperature increases with an increase in the distance from the central axis of the device, which is indicative of the increase in radio frequency heating of the plasma inside a Paul trap.

Secondly, we investigate the dynamics of a single species plasma in a recently proposed dual frequency Paul trap. Such a device is a viable option to trap charged particles of varied charge to mass ratio and offers a flexible confinement of charged particle by keeping the primary voltage and frequency fixed, while varying the secondary voltage and frequency. This changes the form of the pseudo potential well and allows for different spatial regions of confinement. We compare this device with a conventional single frequency Paul trap. The double hump in the time averaged distribution function has a global maxima at $v = 0$ and it seems that a plasma is likely to be more stable in a dual frequency trap than in a conventional single frequency trap. Though the temperature in a dual frequency Paul trap also increases with distance from the central axis of the device, its temporal variation shows an interesting property for periodic solutions of the plasma distribution. If the applied frequencies are rationally related, the temperature oscillates with a time-period given by the LCM of the two time-periods of the applied voltages and their linear combinations. Hence, if the ratio of frequencies is an integer, the plasma temperature oscillations occur at the lower one among these two frequencies. Notably, the systematic shifts in frequency standards on account of second order Doppler and Stark effects are of the same order as that observed for a single frequency Paul trap.

Thirdly, we analyze plasma dynamics in presence of excess micro motion due to stray electric fields and phase mismatch between the electrodes. Since the particles are likely to experience a drift with respect to both position as well as velocity, it becomes important to accommodate such drifts in the initial form of distribution function. The stroboscopic plot reveals a closed curve indicating the existence of a time periodic distribution function in the Paul trap. The instantaneous temperature exhibits an asymmetrical profile quite different from the one obtained in a conventional Paul trap without excess micro motion.

सार

रेडियो आवृत्ति जाल में एकल कण गतिशीलता का व्यापक रूप से कई दशकों तक शोध किया गया है और कई रोचक गुण और अनुप्रयोग सामने आए हैं। हालांकि, एक प्रजाति के एकत्रित कणों की सामूहिक गतिशीलता एक पॉल जाल के साथ-साथ बहुत अच्छी तरह से समझ में नहीं आती है। लागू रेडियो आवृत्ति विद्युत क्षेत्र के कारण प्लाज्मा हीटिंग की घटना की समझ अभी भी अपने बचपन में है। यह थीसिस सैद्धांतिक रूप से पॉल जाल के अंदर एक गैर-तटस्थ प्लाज्मा की गतिशीलता की जांच करता है और प्लाज्मा तापमान भिन्नताओं को समझने के लिए गणितीय ढांचा प्रदान करता है, जो उम्मीदवारों को रेडियो फ्रीक्वेंसी हीटिंग की घटना को बेहतर ढंग से समझने में सहायता करेगा।

सबसे पहले, हम एक पॉल जाल में एक प्रजाति प्लाज्मा का विश्लेषण करते हैं जिसमें अंतरिक्ष चार्ज प्रभावों की उपेक्षा की जाती है और प्लाज्मा को टक्कर-कम माना जाता है। पतली एकल प्रजाति प्लाज्मा के विश्लेषण में लगाए जाने के लिए ये उचित स्थितियां हैं। हम एक विश्लेषणात्मक समय-निर्भर प्लाज्मा वितरण समारोह का निर्माण करके शुरू करते हैं, जो कि क्यू-गॉसियन प्रकार त्सलिस वितरण और स्केल लम्बाई के सावधानी से चुने गए प्रारंभिक रूप के लिए लागू होता है, उसी आवृत्ति के साथ समय-आवधिक होता है जैसे लागू रेडियो आवृत्ति विद्युत क्षेत्र। कण वितरण में बिजली कानून पूछ के अस्तित्व का प्रायोगिक अवलोकन एक Tsallis वितरण चुनने के पीछे प्राथमिक प्रेरणा में से एक है। औसत औसत वितरण समारोह एक निश्चित स्थानिक थ्रेशहोल्ड से परे एक डबल कूल्हे दिखाता है और डबल कूल्हे थोक से दूर चला जाता है क्योंकि त्सलिस पैरामीटर क्यू बढ़ता है, जिससे यह संकेत मिलता है कि मैक्सवेलियन वितरण की तुलना में क्यू-गॉसियन वितरण शायद अधिक स्थिर है क्यू = 1 के साथ त्सलिस वितरण से प्राप्त किया गया है। इसके अलावा, डिवाइस के केंद्रीय धुरी से दूरी में वृद्धि के साथ प्लाज्मा तापमान बढ़ता है, जो कि पॉल जाल के अंदर प्लाज्मा की रेडियो आवृत्ति हीटिंग में वृद्धि का संकेत है।

दूसरा, हम हाल ही में प्रस्तावित दोहरी आवृत्ति पॉल जाल में एक प्रजाति प्लाज्मा की गतिशीलता की जांच करते हैं। इस तरह का एक उपकरण द्रव्यमान अनुपात में विभिन्न चार्ज के चार्ज कणों को फँसाने के लिए एक व्यवहार्य विकल्प है और प्राथमिक वोल्टेज और आवृत्ति को स्थिर रखते हुए चार्ज कण की लचीली बंधन प्रदान करता है, जबकि सेकेंडरी वोल्टेज और आवृत्ति भिन्न होता है। यह छद्म संभावित कुएं के रूप में परिवर्तन करता है और कैद के विभिन्न स्थानिक क्षेत्रों की अनुमति देता है। हम इस डिवाइस की तुलना पारंपरिक पारंपरिक आवृत्ति पॉल जाल से करते हैं। औसत अपरिवर्तनीय फंक्शन में डबल कूल्हे में $v = 0$ पर वैश्विक अधिकतमता होती है और ऐसा लगता है कि पारंपरिक एकल आवृत्ति जाल की तुलना में प्लाज्मा एक दोहरी आवृत्ति जाल में अधिक स्थिर होने की संभावना है। यद्यपि एक दोहरी आवृत्ति में तापमान पॉल जाल भी डिवाइस के केंद्रीय धुरी से दूरी के साथ बढ़ता है, इसके अस्थायी भिन्नता प्लाज्मा वितरण के आवधिक समाधान के लिए एक दिलचस्प प्रोपर्टी दिखाती है। यदि लागू आवृत्तियों तर्कसंगत रूप से संबंधित हैं, तो तापमान लागू वोल्टेज और उनके रैखिक

संयोजनों की दो समयावधि के एलसीएम द्वारा दी गई समयावधि के साथ आता है। इसलिए, यदि आवृत्तियों का अनुपात एक पूर्णांक है, तो इन दो आवृत्तियों के बीच प्लाज्मा तापमान का तापमान निम्न में होता है। विशेष रूप से, दूसरे क्रम के कारण आवृत्ति मानकों में व्यवस्थित बदलाव डोप्लर और स्टार्क प्रभाव उसी क्रम के होते हैं जैसा कि एक आवृत्ति पॉल जाल के लिए मनाया जाता है।

तीसरा, हम विद्युत विद्युत क्षेत्रों और इलेक्ट्रोड के बीच चरण विसंगति के कारण अतिरिक्त सूक्ष्म गति की उपस्थिति में प्लाज्मा गतिशीलता का विश्लेषण करते हैं। चूंकि पक्षियों को दोनों पदों के साथ-साथ वेग के संबंध में बहाव का अनुभव होने की संभावना है, इसलिए वितरण कार्य के शुरुआती रूप में ऐसी बहाव को समायोजित करना महत्वपूर्ण हो जाता है। स्ट्रोबोस्कोपिक प्लॉट पॉल ट्रैप में एक समय आवधिक वितरण समारोह के अस्तित्व को दर्शाते हुए एक बंद वक्र का खुलासा करता है। तात्कालिक तापमान अतिरिक्त सूक्ष्म गति के बिना एक संयोजक पॉल जाल में प्राप्त एक से एक विषम प्रोफ़ाइल प्रदर्शित करता है।

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