ANALYTICAL SOLUTIONS FOR MICRO- AND MACRO-MECHANICS OF SMART STRUCTURES INTEGRATED WITH PIEZOELECTRIC COMPOSITES

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by

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Dedicated to
my mother (Omwati Devi) and father (Dharampal Singh) whose thoughts of aiming high have always been my source of inspiration

Certificate

This is to certify that the thesis entitled "**Analytical Solutions for Micro- and Macro-Mechanics of Smart Structures Integrated with Piezoelectric Composites**" being submitted by **Ms. Poonam Kumari** to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by her under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Poonam Kumari

Abstract

Composite and sandwich laminates with embedded or surface-mounted piezoelectric sensors and actuators form a very important part of the new generation of smart structures for active vibration control, acoustic control, shape control, damage detection and health monitoring applications. Due to the presence of high layerwise inhomogeneity in mechanical, thermal and electric properties in these hybrid laminates, modeling of structures of such laminates requires special attention. The most accurate global as well as local layerwise response of hybrid laminated structures can be obtained by the exact analytical solution of the equations of three dimensional (3D) piezoelasticity. Recent developments of the piezoelectric fiber reinforced composite (PFRC) with high strength, toughness, operating range and life, and conformability to curved shell surfaces have widened the scope of use of distributed piezoelectric sensors and actuators in smart structures. The prediction of response of hybrid laminated structures integrated with PFRC layers requires an accurate determination of their effective properties.

In this thesis, a fully coupled 3D micromechanical model is presented for computing the effective electrothermoelastic properties of a unidirectional PFRC lamina with the poling and electric field applied normal to the fiber direction. Using the isofield method, the effective properties are obtained for representative volume elements (RVEs) of two possible connectivities of the piezoelectric and matrix phases, which are then combined. The model considers differential electric fields in fiber and matrix phases due to their different dielectric constants. The effects of fiber volume fraction v_f and dielectric ratio (DR) (piezoelectric fiber to matrix) on the effective properties are studied for two PFRC systems.

The powerful extended Kantorovich method (EKM) originally proposed by Kerr (1968) and so far used only for two-dimensional (2D) elasticity problems is generalized first to the 3D elasticity solution for multilayered plates in cylindrical bending, and then to 3D piezoelastic-

ity solution for the coupled electromechanical response of hybrid plates integrated with PFRC sensor/actuator layers. The significant extensions made to the method in this process are (1) the application to the 3D elasticity and piezoelasticity problems involving an inplane direction and a thickness direction instead of both inplane directions in 2D elasticity problems, (2) the treatment of the nonhomogeneous boundary conditions encountered in the thickness direction, and (3) the use of a mixed variational principle to obtain the governing differential equations in terms of displacements/electric potential as well as stresses/electric displacements. The mixed formulation allows for exact satisfaction of boundary conditions at all points, and also ensures the same order of accuracy for variables. The convergence and accuracy of the method are established for both pressure and electric potential loadings in comparison with other 3D analytical solutions, wherever available, and else with the 3D finite element (FE) solution. The inconsistencies and pitfalls of the FE solution at or very near to the supports are pointed out. The present study will facilitate development of accurate semi-analytical solutions of many other unresolved problems in 3D piezoelasticity such as the free edge stresses in hybrid piezoelectric laminates, responsible for delamination failure.

Benchmark analytical 3D piezoelasticity solutions for the static and dynamic response of simply-supported hybrid cylindrical shells of revolution and shell panels integrated with monolithic piezoelectric or PFRC layers are also presented. The piezoelectric layers are polarized along the radial direction to induce extension actuation/sensing mechanism. The solutions presented are valid for shallow as well as deep shell panels. Results for the natural frequencies and steady state response due to harmonic electromechanical excitation are presented for single layer piezoelectric panels as well as multilayered hybrid composite and sandwich panels. For PFRC laminated shell panels, the effects of the PFRC fiber orientation, dielectric ratio of the fiber and matrix phases and the fiber volume fraction are studied for the static response under electrothermomechanical loading and vibration response under electromechanical loading. In addition, analytical solutions for Levy-type rectangular hybrid plates based on a recently developed coupled efficient layerwise 2D theory, called the zigzag theory, and its smeared counterpart, the third order theory, are presented so as to study the boundary layer effects as well as assessing their accuracy in comparison with 3D piezoelasticity solutions. This constitutes the first analytical solution for Levy-type smart plates, which considers the two-way electromechanical coupling. The effect of electromechanical coupling on the strength of boundary layer is studied.

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