

**GROWTH-INDUCED INSTABILITY AND  
MORPHOLOGICAL CHANGES IN HIGHLY  
FLEXIBLE SLENDER STRUCTURES**

**ROUSHAN KUMAR**



**DEPARTMENT OF APPLIED MECHANICS  
INDIAN INSTITUTE OF TECHNOLOGY DELHI  
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FLEXIBLE SLENDER STRUCTURES**

by

**ROUSHAN KUMAR**

Department of Applied Mechanics

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



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*Dedicated to my beloved parents*

# Certificate

This is to certify that the thesis entitled “**Growth-Induced Instability and Morphological Changes in Highly Flexible Slender Structures**”, being submitted by **Mr. Roushan Kumar** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy**, is a record of bonafide research carried out by him under my supervision. The thesis, in my opinion, merits evaluation in accordance with the Institute’s rules and regulations. To the best of my knowledge and belief, the content of this thesis has not been submitted in full or in part to any other university or institute to confer any degree or diploma.

Dr. Ajeet Kumar  
Professor  
Department of Applied Mechanics  
Indian Institute of Technology Delhi  
New Delhi-110016  
India

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# Abstract

Slender structures such as rods and plates are ubiquitous in both natural and engineered systems. Their highly flexible nature makes them prone to large deformations, geometric nonlinearities, instabilities, and post-buckling responses, which play a crucial role in determining their mechanical behavior. These phenomena are central to a wide range of applications, including soft robotics, flexible electronics, and biological growth processes. However, traditional elasticity theories for modeling such structures often fall short in describing such systems, as they are typically restricted to small deformations and cannot adequately capture the geometric nonlinearities, finite rotations, and intrinsic growth effects that govern their behavior. To overcome these limitations, generalized continuum theories—most notably the Cosserat rod and plate models—have been developed. These models provide a versatile framework that naturally incorporates shear, finite rotations, and complex boundary interactions, allowing for an accurate description of three-dimensional kinematics using one- or two-dimensional continua. Building upon these foundations, recent advances in morphoelasticity have further extended the theoretical scope to include growth and active material responses, thereby enabling the study of growth-driven mechanics in biological and bioinspired systems. This thesis aims to develop non-linear Cosserat-based models for highly flexible slender structures, with particular emphasis on growth-induced deformations.

The first part of this thesis develops a nonlinear constitutive model to determine the natural shapes and stiffnesses of morphoelastic rod-like structures modeled as special Cosserat rods. The formulation allows the rod's cross-section to have an arbitrary shape, and the material to undergo both volumetric and surface growth. The helical Cauchy–Born rule is employed to generate a helical rod in which the strain parameters are imposed uniformly along its arc length. This uniformity in the strain field enables the reduction of the three-dimensional nonlinear equations of morphoelasticity from the entire rod body to its cross-sectional plane. The governing equations for the deformation of the rod's cross-section are then derived by considering both volume and surface growth. Semi-analytical expressions for the stress resultants and stiffnesses of the rod are subsequently obtained in terms of the prescribed strains, growth parameters, and the unknown deformed cross-section. By setting the stress resultants to zero, the natural shape of the rod is determined. A finite element formulation is also presented to solve the non-linear cross-sectional morphoelasticity problem. To elucidate the presented computational framework, several numerical simulations are performed involving volumetric uniform and differential axial growth, isotropic surface growth and active filaments.

The second part of this thesis develops a one-dimensional constitutive theory for ribbons or strip-like structures, modeled as special Cosserat rods through a reduced formulation derived from a general Cosserat plate theory. Starting with the description of a strip as a general Cosserat plate, the strip is subjected to a strain field that is uniform along its length. The helical Cauchy–Born rule is employed to impose this uniform strain field, deforming the strip into a six-parameter family of helical configurations, where the six parameters correspond to the six strain measures of rod theory. Two vector variables are introduced: one to describe the position of the deformed centerline of the strip’s cross-section, and the other to characterize the orientation of the thickness lines along the strip’s width. The minimization of the strip’s plate energy under the constraint of the aforementioned uniformity in strain field reduces the partial differential equations of plate theory from the entire mid-plane of the strip to just a system of nonlinear ordinary differential equations along the strip’s width line for the above mentioned two vector variables. A nonlinear finite element formulation is further presented to solve this system of equations. This formulation yields the strip’s stored energy per unit length, along with the induced internal forces, moments, and stiffnesses for any prescribed set of six strain measures from rod theory. The proposed scheme is applied to study uniform bending, twisting, and shearing of strips. In the cases of uniform twisting and shearing, the strip is observed to buckle along its width into more complex configurations, which are accurately captured by the present formulation. The results demonstrate that the proposed scheme is more general and accurate than the existing approaches available in the literature.

Finally, the third part of this thesis presents a comprehensive theoretical and numerical framework for modeling and analyzing the growth-induced deformation and instability behavior of a coupled plate–rod system. The governing equations and boundary conditions of a special Cosserat circular plate bounded by a special Cosserat rod are systematically derived using variational principles from the total energy of the coupled system. A finite element formulation is also presented to solve the non-linear equilibrium equations of a growing system. Symmetry-reduced boundary conditions are introduced to efficiently capture post-buckling configurations. Analytical formulations are then presented to determine the critical growth parameter corresponding to both axisymmetric and planar non-axisymmetric deformation modes. Special cases, including Kirchhoff-type plate and rod combinations, are examined to provide simplified closed-form expressions for the buckling thresholds. The study further explores the post-buckling response of the coupled system through numerical simulations, revealing transitions from axisymmetric to planar multi-lobed morphologies. Finally, comparisons among various coupled models, ranging from Cosserat to von Kármán formulations, highlight the influence of kinematic and constitutive assumptions, as well as plate and rod stiffness, on the onset and evolution of buckling instabilities under growth.

## सारांश

पतले संरचनात्मक तंत्र जैसे रॉड और प्लेट प्राकृतिक तथा अभियांत्रिक दोनों प्रकार की प्रणालियों में व्यापक रूप से पाए जाते हैं। इनकी अत्यधिक लचीली प्रकृति इन्हें बड़े विकृतियों, ज्यामितीय अरेखिकताएँ, अस्थिरताओं तथा पोस्ट बकलिंग प्रतिक्रियाओं के प्रति संवेदनशील बनाती है, जो इनके यांत्रिक व्यवहार को निर्धारित करने में महत्वपूर्ण भूमिका निभाते हैं। ये घटनाएँ सॉफ्ट रोबोटिक्स, लचीली इलेक्ट्रॉनिक्स तथा जैविक वृद्धि प्रक्रियाओं सहित अनेक अनुप्रयोगों में केंद्रीय महत्व रखती हैं। परंतु पारंपरिक प्रत्यास्थता सिद्धांत, जो ऐसी संरचनाओं के मॉडलन के लिए प्रयुक्त होते हैं, प्रायः छोटे विकृतियों तक सीमित रहते हैं और ज्यामितीय अरेखिकताओं, सीमित घूर्णनों तथा अंतर्निहित वृद्धि प्रभावों को समुचित रूप से अभिव्यक्त नहीं कर पाते। इन सीमाओं को दूर करने के लिए सामान्यीकृत सतत माध्यम सिद्धांत—विशेषकर कोसेरात रॉड तथा प्लेट मॉडल—विकसित किए गए हैं। ये मॉडल एक बहुउद्देशीय रूपरेखा प्रदान करते हैं, जिसमें शीयर विकृति, सीमित घूर्णन तथा जटिल सीमांत अंतःक्रियाएँ स्वाभाविक रूप से सम्मिलित होती हैं, जिससे एक या द्वि-आयामी सतत माध्यम के माध्यम से त्रि-आयामी काइनेमैटिक्स का सटीक वर्णन संभव होता है। इन आधारों पर निर्मित हालिया प्रगतियों में मोर्फो-प्रत्यास्थता ने वृद्धि तथा सक्रिय पदार्थ प्रतिक्रियाओं को सम्मिलित करते हुए जैविक एवं जैव-प्रेरित प्रणालियों में वृद्धि-चालित यांत्रिकी के अध्ययन का मार्ग प्रशस्त किया है। इस शोधप्रबंध का उद्देश्य अत्यधिक लचीली पतली संरचनाओं के लिए वृद्धि-प्रेरित विकृतियों पर विशेष बल देते हुए अरेखिक कोसेरात-आधारित मॉडल विकसित करना है।

इस शोधप्रबंध के प्रथम भाग में विशेष कोसेरात रॉड के रूप में प्रतिरूपित मोर्फो-प्रत्यास्थ रॉड-सदृश संरचनाओं के प्राकृतिक आकार तथा कठोरताओं के निर्धारण हेतु एक अरेखिक संरचनात्मक मॉडल विकसित किया गया है। इस रूपरेखा में रॉड का अनुप्रस्थ काट मनमाने आकार का हो सकता है तथा पदार्थ में आयतनात्मक एवं सतही दोनों प्रकार की वृद्धि संभव है। हेलिकल कौशी-बोर्न नियम का उपयोग कर रॉड में समान रूप से वितरित विकृति क्षेत्र आरोपित किया गया है, जिससे त्रि-आयामी अरेखिक मोर्फो-प्रत्यास्थ समीकरणों को संपूर्ण रॉड से घटाकर केवल उसके अनुप्रस्थ काट के तल तक सीमित किया जा सके। आयतन तथा सतही वृद्धि को ध्यान में रखते हुए अनुप्रस्थ काट के विकृति समीकरण व्युत्पन्न किए गए हैं। तत्पश्चात, आरोपित विकृतियों, वृद्धि मानकों तथा अज्ञात विकृत अनुप्रस्थ काट के फलन के रूप में आंतरिक बलों तथा कठोरताओं के अर्ध-विश्लेषणात्मक व्यंजक प्राप्त किए गए हैं। आंतरिक बलों को शून्य करने पर रॉड का प्राकृतिक आकार निर्धारित होता है। अरेखिक अनुप्रस्थ मोर्फो-प्रत्यास्थ समस्या के समाधान हेतु एक सीमित अवयव विधि भी प्रस्तुत की गई है। इस संगणनात्मक रूपरेखा को स्पष्ट करने के लिए आयतनात्मक समरूप तथा अवकल अक्षीय वृद्धि, समदिश सतही वृद्धि तथा सक्रिय तंतुओं से संबंधित अनेक संख्यात्मक अनुकरण किए गए हैं।

द्वितीय भाग में रिबन अथवा पट्टी-सदृश संरचनाओं के लिए एक आयामी संरचनात्मक सिद्धांत विकसित किया गया है, जिसे सामान्य कोसेरात प्लेट सिद्धांत से व्युत्पन्न एक अपसारित रूपरेखा के माध्यम से विशेष कोसेरात रॉड के रूप में प्रतिरूपित किया गया है। पट्टी को एक सामान्य कोसेरात प्लेट के रूप में निरूपित करते हुए उस पर लंबाई के साथ समान विकृति क्षेत्र आरोपित किया गया है। हेलिकल कौशी-बोर्न नियम के माध्यम से पट्टी को छह-परिमितीय हेलिकल विन्यासों के परिवार में विकृत किया जाता है, जहाँ ये छह परिमाण रॉड सिद्धांत की छह विकृति मात्राओं के अनुरूप हैं। दो सदिश चर प्रस्तुत किए गए हैं—एक विकृत केंद्रेखा की स्थिति के लिए तथा दूसरा चौड़ाई दिशा में मोटाई रेखाओं की अभिविन्यास के लिए। आरोपित समान विकृति क्षेत्र की बाधा के अधीन पट्टी की प्लेट ऊर्जा के न्यूनकरण से प्लेट सिद्धांत के आंशिक अवकल समीकरणों को घटाकर केवल

चौड़ाई रेखा के अनुदिश अरेखीय साधारण अवकल समीकरणों के एक तंत्र में परिवर्तित किया गया है। इस तंत्र के समाधान हेतु अरेखीय सीमित अवयव रूपरेखा प्रस्तुत की गई है, जो प्रति इकाई लंबाई संग्रहीत ऊर्जा, आंतरिक बल, आघूर्ण तथा कठोरताओं का निर्धारण करती है। इस पद्धति का उपयोग समान वक्रण, मरोड़ तथा शीयर विकृति के अध्ययन में किया गया है। विशेषतः मरोड़ तथा शीयर की स्थिति में पट्टी अपनी चौड़ाई के अनुदिश जटिल विन्यासों में विकृत होती है, जिन्हें यह रूपरेखा सटीक रूप से अभिव्यक्त करती है। परिणाम दर्शाते हैं कि यह पद्धति साहित्य में उपलब्ध मौजूदा दृष्टिकोणों की तुलना में अधिक सामान्य तथा अधिक सटीक है।

अंततः तृतीय भाग में वृद्धि-प्रेरित विकृति तथा अस्थिरता व्यवहार के अध्ययन हेतु एक संयुक्त प्लेट-रॉड तंत्र के लिए एक व्यापक सैद्धांतिक एवं संगणनात्मक रूपरेखा प्रस्तुत की गई है। विशेष कोसेरात वृत्ताकार प्लेट, जो विशेष कोसेरात रॉड से सीमाबद्ध है, के लिए युग्मित तंत्र की कुल ऊर्जा से परिवर्तनीय सिद्धांतों के माध्यम से शासक समीकरण तथा सीमांत शर्तें व्युत्पन्न की गई हैं। वृद्धि-युक्त तंत्र के अरेखीय संतुलन समीकरणों के समाधान हेतु सीमित अवयव रूपरेखा प्रस्तुत की गई है। पोस्ट बकलिंग विन्यासों को कुशलतापूर्वक अभिव्यक्त करने के लिए सममिति-अपसारित सीमांत शर्तें लागू की गई हैं। अक्षसममित तथा समतलीय अन-अक्षसममित विकृति विधियों के लिए महत्वपूर्ण वृद्धि मानकों के विश्लेषणात्मक व्यंजक प्राप्त किए गए हैं। किर्खहॉफ-प्रकार की प्लेट-रॉड युग्मन जैसी विशेष स्थितियों में वक्रण सीमा के लिए सरल बंद-रूप व्यंजक प्राप्त किए गए हैं। संख्यात्मक अनुकरणों के माध्यम से पोस्ट बकलिंग प्रतिक्रियाओं का विश्लेषण किया गया है, जिसमें अक्षसममित से बहु-लोबीय समतलीय आकृतियों की ओर संक्रमण प्रदर्शित होता है। अंततः कोसेरात तथा वॉन कार्मान रूपरेखाओं सहित विभिन्न युग्मित मॉडलों की तुलना द्वारा यह दर्शाया गया है कि काइनेमैटिक तथा संरचनात्मक मान्यताओं, तथा प्लेट एवं रॉड की कठोरताओं का वृद्धि-प्रेरित वक्रण अस्थिरताओं के प्रारंभ तथा विकास पर महत्वपूर्ण प्रभाव पड़ता है।

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