

CODE SEQUENCES FOR SPREAD SPECTRUM MULTIPLE ACCESS AND CRYPTOGRAPHIC APPLICATIONS

BY

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CERTIFICATE

This is to certify that the thesis entitled "**Code Sequences for Spread Spectrum Multiple Access and Cryptographic Applications**", being submitted by P.V.A.Narasimham to the Department of Electrical Engineering, Indian Institute of Technology, New Delhi, for the award of degree of Doctor of Philosophy, is a bonafide research work carried out by him under our supervision and guidance. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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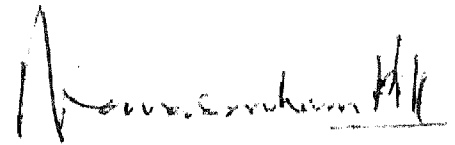
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Abstract

Spread spectrum multiple access (SSMA) methods are generally used in satellite communication systems to combat jamming as well as to protect sensitive information from eavesdropping etc. The presently used SSMA systems can be broadly classified into two categories, viz., the direct sequence systems and the frequency hopping systems. Both these require the use of code sequences with certain kinds of correlation and complexity properties. Code division multiplexing is the important mechanism of multiplexing in these systems. Thus the spreading sequences or codes are required to exhibit low auto-correlation and cross-correlation sidelobes to minimize cross-interference. The families of Gold and Gold-like sequences, Kasami sequences and several others exhibit these properties and are commonly used. Further, in order to make eavesdropping difficult, it is desirable that the sequences should have a *large equivalent linear span*. Finally, to accommodate a large number of users, it is required that the cardinality or the size of the set containing the desirable kind of code words be large. Very few of the presently known codes satisfy all these requirements simultaneously. Similarly, there are very few known classes of frequency hopping patterns which combine the requirements of the so-called optimal Hamming correlation properties with high linear span.

An attempt has been made in this thesis to introduce many new classes of code sequences for the spread spectrum multiple access systems with a cryptographic flavor. More specifically, we have attempted here to derive techniques which can yield all the three desired features mentioned above, in the designed code sequence families. It is demonstrated that bit interleaving techniques which can be defined around certain known families of code sequences can be used effectively to generate additional classes of sequences which exhibit many of the desired properties.

To start with, a new family of large sequence sets, called here the *Narasimham-Prasad sequences* have been introduced. These are obtained by cyclically combining different phases of an m-sequence with certain related elementary sequences, generated by an irreducible polynomial. The sequence sets of this class are shown to have peak values of the CCF and out of phase ACF which are of the same order (being in fact, somewhat lower) as those of the famous Gold and Gold-like sequence sets. While the linear span of these sequence sets is not high, these are seen to exhibit an interleaved

structure which is exploited subsequently to generate additional families of sequences with both good correlation as well as linear span properties.

The use of bit-interleaving for synthesis or construction of sequences has received passing treatment in the literature. This technique, however, is potentially very powerful, and in this work we have attempted to demonstrate this fact. The use of this technique in the present work has been primarily motivated by the following two observations: (a) certain well known sequence classes have an interleaved structure in terms of smaller m-sequences and (b) the cross-correlation properties of such interleaved sequence sets are only dependent on the number of certain null subsequences that arise in the modulo-two sum of the member sequences. Further, these two properties are independent of the actual m-sequence used in the interleaved representation. These general findings have made it possible for us to introduce *three* classes of interleaved sequence sets, viz., the so-called *Kasami-like sequence sets*, the *Niho-like sequence sets* and the *Narasimham-Prasad like sequence sets*, which are derived, as the name suggests, from the parent sequences of the corresponding types. The new sequence families are shown to share the correlation properties of the corresponding parent families. The linear complexities of all the three classes of sequence sets, however, are seen to be very high as compared to those of the parent sequence classes. An important additional advantage offered by this construction technique is that many such sets of sequences can be obtained for each set of the parent class.

Later, we generalize this construction procedure into an even more powerful concept of *level-k interleaving*. With the help of this generalized concept it is possible to get phenomenal increase in the density of optimal sequence sets, along with significant enhancements in the values of their linear span.

The extension of interleaving techniques for the generation of optimal frequency hopping patterns for the frequency hopped spread spectrum systems is next investigated. It is shown that in combination with the concept of σ_k -transforms, the process of interleaving can now be used to generate additional optimal families of frequency hopping patterns starting from m-sequences and m-like sequences. Once again, it is possible to obtain several sequence sets starting from each parent set of a known family of such patterns.

Finally, we discuss a few miscellaneous sequence sets and their properties. The discussion on the interleaved structure of m-sequences and m-like sequences, which form the basis of much of the work on the thesis, is concluded with some thoughts on further possible manipulations in the interleaved structure. It is shown, for example that inversion of the null subsequences in certain subclass of m-like sequences can yield a considerable improvement in the linear span of these at a marginal cost in terms of their ACF properties (the ACF sidelobe values of "-1" are replaced with sidelobe values of "-1" and "3"). Similar properties are shown to be available for similar modifications on the level-k m-like sequences.

Table of Contents

Acknowledgements	i
Abstract	ii
List of Abbreviations	ix
List of Symbols	x
List of Figures	xv
List of Tables	xvi
 Chapter I Introduction to the Thesis	 1
1.1 Introduction	2
1.2 Background	2
1.2.1 Definitions and Preliminaries	2
1.2.2 Discussion	6
1.3 A Brief Survey of the Previous Work	7
1.3.1 Properties of m-sequences	7
1.3.2 Bounds on Peak Correlation Function Values	12
1.3.3 A Class of Composite m-sequences	13
1.3.4 Cross-correlation between m-sequences and m-subsequences	14
1.3.5 Correlation Properties of the Elementary Sequences	14
1.3.6 Binary Sequence Sets derived from m-sequences	15
1.3.7 Frequency Hopping Systems	26
1.4 Scope of the Thesis	33
1.4.1 Construction via Combining Sequences with an Interleaved Structure	35
1.4.2 A Non-linear Construction Method	35
1.4.3 Frequency Hopping Patterns	36
1.4.4 Miscellaneous Sequence Sets	36
1.5 Brief Outline of the Thesis	37
 Chapter II A New Class of Sequences with Four Valued Auto- and Cross-correlation Functions	 39
2.1 Introduction	40

2.2	A Brief Review of Elementary Sequences and their Properties	41
2.3	Definition and Properties of Trace Functions	43
2.4	Definition and Construction of the New Class of Sequence Sets	45
2.5	Correlation Properties	48
2.6	Equivalent Linear Span of the New Class of Sequence Sets	53
2.7	Remarks and Conclusions	54
Chapter III	Interleaved Sequences with Good Correlation Properties and High Linear Complexity	55
3.1	Introduction	56
3.2	Cross-correlation Properties of Interleaved Sequences	56
3.3	A New Optimal Class of Sequences (Kasami-like Sequences)	60
3.3.1	An Interleaved Representation of m-sequences	60
3.3.2	Representation of Kasami Sequences as a Linear Combinations of Interleaved m-sequences	62
3.3.3	Kasami-like Sequences	63
3.4	Large Sets of Sequences Based on Interleaving Representation	68
3.4.1	Niho-like Sequences	68
3.4.2	Another Large Set of Sequences	70
3.5	The Equivalent Linear Complexity of the New Classes of Sequences	70
3.6	Level-k Interleaving and its Applications	78
3.7	Conclusions	87
Chapter IV	New Frequency Hopping Patterns with Optimal Hamming Correlation Properties	88
4.1	Introduction	89
4.2	Construction of a New Class of Frequency Hopping Patterns	90
4.3	Hamming Correlation Properties and Optimality of the Classes K and K' of Frequency Hopping Patterns	96
4.3.1	Hamming Correlation Properties	96
4.3.2	Optimality of the Pattern Classes K and K'	101
4.4	Remarks and Comparisons with other Frequency Hopping Patterns	102
4.4.1	Equivalent Linear Complexity	102

4.4.2 Comparison	102
4.4.3 Additional Sets of Frequency Hopping Patterns	104
4.5 Conclusions	104
Chapter V Miscellaneous New Sequences and Sequences Sets for some SSMA Applications	105
5.1 Introduction	106
5.2 Modified m-like Sequences	106
5.2.1 Definition and Construction	107
5.2.2 Auto-correlation Properties	107
5.2.3 Linear Complexity of the Modified m-like Sequences	108
5.2.4 Remarks	109
5.3 Some Additional Sequence Classes for Secure Multiple Access Systems	111
5.3.1 Definition and Construction	112
5.3.2 Derivation of Auto-correlation Functions and Typical Values of ACF	112
5.3.3 Cross Correlation Properties	118
5.3.4 Linear Complexity Analysis	118
5.3.5 Comparison of the Classes C and D with the Gold sequence sets	119
5.4 Conclusions	122
Chapter VI Conclusions and Suggestions for Further Work	123
6.1 Introduction	124
6.2 Summary of Results	124
6.3 Suggestions for Further Work	127
Appendix I Interleaved Representation of u and $\{v^k, k=0,1,2\}$ in terms of a Common m_1-subsequence w	129
Appendix II Derivation of Eqns. (3.3a) and (3.3b)	132

Appendix III Bounds on Equivalent Linear Span of m-like Sequences and Modified m-like Sequences	135
A.3-1 Introduction	136
A.3-2 Equivalent Linear Span of the m-like sequences and Modified m-like sequences	136
A.3-3 Equivalent Linear Polynomials and Their Factorization	137
References	144
Bio-data	148