

**COMPUTATIONAL INVESTIGATION OF TURBULENT
SLOT JET IMPINGEMENT HEAT TRANSFER USING LES
AND RANS MODELLING**

by

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CERTIFICATE

This is to certify that the thesis entitled “Computational Investigation of Turbulent Slot Jet Impingement Heat Transfer using LES and RANS Modeling” being submitted by Rabijit Dutta is the report of bonafide research work carried by him under our supervision. This thesis has been prepared in conformity with the rules and regulations of INDIAN INSTITUTE OF TECHNOLOGY New Delhi. We further certify that the thesis has attained a standard required for the award of a Ph.D degree of the Institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

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ABSTRACT

Jet impingement is a flow configuration which is encountered in various practical applications because of its high heat and mass transfer rates. Jet impingement has a complicated fluid flow and heat transfer behaviours which largely depend on the distance from the nozzle exit to the impinged plate. A fundamental understanding of these behaviours is essential for developing turbulence models which can accurately predict jet impinging flows. Further, it is vital to find out ways to enhance the heat transfer from jet impinging flows.

In the present thesis large eddy simulation (LES) and Reynolds-averaged Navier-Stokes (RANS) equation based computations have been performed for slot jet impingement heat transfer. Computations have been performed both at a small nozzle-to-plate spacing and at a high nozzle-to-plate spacing at practical values of Reynolds numbers. Performances of a large number of RANS models have been assessed in the present study by comparing the computations with the experimental data. Further, LES computations have been performed to better understand the flow and heat transfer characteristics and to develop improved RANS models for jet impingement flow. Finally, the heat transfer enhancement from jet impinging flows has been studied by employing nanofluids.

Different versions of k - ϵ , k - ω and v^2 - f and Spalart-Allmaras (SA) models have been studied for the non-dimensional nozzle-to-plate spacings (H/B) of 4 and 9.2 and the mean velocity, turbulence fluctuations and surface Nusselt number have been compared with the corresponding experimental data reported in the literature. It was observed that the models which can capture the flow transition behaviour, namely, SA model and k - ω model with transition corrections perform better in predicting the secondary peak in the Nusselt number at the small nozzle-to-plate spacing (H/B) of 4. Further, it was observed that the standard k - ϵ model, standard k - ω model and the SA model correctly predicted the Nusselt number distribution at H/B of 9.2. Some of the models show a false secondary peak which was due to an under-prediction of the turbulence kinetic energy in the vicinity of the stagnation region.

LES computations of slot jet impingement have been performed at H/B of 4 and 10 at Reynolds numbers of 20000 and 13500, respectively. The well-known flow structures in the stagnation region, viz., counter-rotating vortices have been visualized from the LES at H/B of 10. Further, LES at H/B of 4 shows that both the streamwise and the wall normal turbulence increases suddenly near the location of the secondary Nusselt number peak which could be attributed to the appearance of large vortices that are convected from the jet shear layer. Furthermore, the prediction of the streamwise velocity profiles from the LES shows that it does not follow the law-of-the-wall in the wall jet region and the deviation is large near the stagnation region.

LES results have been employed for a deeper understanding of the performance of RANS models for slot jet impingement heat transfer. Comparisons of RANS and LES predictions have been performed in terms of various turbulence quantities to assess these models. Further, a framework has been developed to improve RANS models using the LES data for slot jet impingement heat transfer. This is done by using LES predicted data for a priori predictions of RANS model turbulent viscosity and the model coefficients. The standard k- ϵ model has been studied using this framework and it is found that by specifying an equation for the coefficient c_μ the model can be improved for the prediction of slot jet impinging flows.

Finally, enhancement of heat transfer using Al₂O₃-water nanofluids for laminar and turbulent slot jet impingement heat transfer has been studied. Comparisons with different values of volume fractions of nanofluids at constant Reynolds number show that although heat transfer increases with the employment of nanofluids, the pressure drop rises drastically which leads to an overall decrease in the energy based performance measured in terms of performance evaluation criterion (PEC). Further, a comparison at a constant velocity shows that the heat transfers from the impingement surface using nanofluids are less than that using the base fluid (water).

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NOMENCLATURE

B = slot jet width, m

c_p = specific heat of the working fluid, J/kg-K

f = elliptic relaxation parameter

h = surface heat transfer coefficient, $h = \frac{\lambda \frac{\partial T}{\partial y}|_{wall}}{(T_w - T_o)}$, W/m²-K

H = distance between nozzle exit to impingement plate, m

\bar{h} = average heat transfer coefficient, $\bar{h} = \frac{1}{l} \int_0^l h dx$, W/m²-K

h_r = ratio of average heat transfer coefficient between nanofluid and base fluid, $h_r = \frac{\bar{h}_{nf}}{\bar{h}_{bf}}$

Nu = Nusselt number at the wall, $Nu = \frac{hB}{\lambda}$

PP_r = ratio of pumping power between nanofluid and the base fluid, $PP_r = \frac{PP_{nf}}{PP_{bf}}$

Pr = Prandtl number, $Pr = \frac{\mu_{nf} c_{p,nf}}{\lambda_{nf}}$

Pr_t = turbulent Prandtl number

Re = Reynolds number, $Re = \frac{\rho_{nf} V_0 B}{\mu_{nf}}$

T = static temperature, K

u_i = velocity vector, m/s

$\overline{v^2}$ = wall-normal fluctuating velocity variance, m²/s²

V_0 = wall-normal velocity at the jet inlet, m/s

\dot{V} = volume flow rate, m³/s

Δp_t = difference between the average total pressure at the inlet and the outlet, Pa

Greek Symbols

φ = volume Fraction, $\varphi = \frac{V_{nf}}{V}$

k = turbulence kinetic energy, m²/s²

ε = turbulent dissipation rate, m²/s³

ω = specific dissipation rate, 1/s

ρ = density of the working fluid, kg/m³

μ = dynamic Viscosity, kg/m-s
 μ_t = turbulent viscosity, kg/m-s
 λ = thermal Conductivity, W/m-K

Subscript

i,j = index of coordinate direction
0 = quantities at the inlet
nf = quantities related to nanofluid
bf = quantities related to base fluid (water)
p = quantities related to nanoparticles
avg = average
0 = related to stagnation zone
ENH = related to enhanced wall treatment

Superscript

' = fluctuating quantity
* = related to the near wall region
- = time averaged quantity

Abbreviation

SST = shear stress transport used in Section 3.2.1
PEC = performance evaluation criteria, $PEC = \frac{\dot{m}C_p\Delta T}{\dot{V}\Delta p_t}$
PP = pumping power, $PP = \dot{V}\Delta p_t$, W
LES= large eddy simulation
RANS=Reynolds-averaged Navier Stokes
SGS=sub-grid stress
DES=detached eddy simulation.