

**IMPACT RESPONSE OF LARGE SPECIALLY
ORTHOTROPIC LAMINATES WITH
SHEAR DEFORMATION**

by

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CERTIFICATE

This is to certify that the thesis entitled **“IMPACT RESPONSE OF LARGE SPECIALLY ORTHOTROPIC LAMINATES WITH SHEAR DEFORMATION”** being submitted by **Mr. Muhammad Shamasuddeen Jafiri** to the Indian Institute of Technology, Delhi for the award of the Degree of **DOCTOR of PHILOSOPHY** is a record of bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the standard fulfilling the requirements for the award of the degree of **Doctor of Philosophy**.

The research report and the results presented in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.



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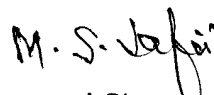
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ABSTRACT

Fibre-reinforced composite materials, because of having high strength-to-weight and stiffness-to-weight ratios, are being increasingly used in applications where weight saving is an important criterion, such as in load bearing structural components of aircrafts and space vehicles etc. The behaviour of these structures is not satisfactory under impact loads and hence it is imperative to predict the response of such structures to impact loads which may be due to a falling weight or a projectile hit etc. The response of fibre-reinforced composite structures to static or quasi-static loads is well understood and documented. Also their behaviour under free and forced vibrations has been investigated in detail. However, little attention has been paid to the problem of transverse impact on beams and plates of fibre-reinforced composite materials. The impact response generally deteriorates with increase in fibrous reinforcement. An impact event involves relatively high contact forces acting on a small area and over a short period of time. High stresses, elastic and plastic deformations and many fracture events occur during impact.

For maximum efficiency of reinforcement, these materials are used in the form of aligned fibre beams, plates or shells. Such an arrangement is generally termed as a lamina and it obviously exhibits orthotropic elastic behaviour. A lamina has high strength in the direction of fibres and poor strength perpendicular to the fibres and therefore it can not be directly used in any structure subjected to impact loading. However, in more practical situations, particularly when a multi-axial stress field is expected in a structure, an assembly of laminas termed as laminate is used. Several laminas are bonded

together and cured to form a laminate, in which the properties and the orientation of laminae are chosen to meet the design requirements. In either of the two forms, the stress-strain relationship for the material is generally linear almost upto failure in case of a lamina and upto first ply failure in case of a laminate.

The present work deals with the dynamic response under transverse impact of a large orthotropic linearly elastic plate (lamina/laminate) with aligned continuous fibres. The behaviour of a lamina and a laminate under certain restrictions of symmetry has been described without and with shear deformations.

The impact load considered in this analysis need not be concentrated at a point, as has been assumed in most of the existing literature. Instead the impact load distribution is based on Sveklo's theory of contact between anisotropic bodies which is a modification of the classical Hertz's theory for isotropic bodies. Accordingly, the force is considered to be distributed in a parabolic manner over an elliptical area, whose size depends on the magnitude of the impact load and the elastic properties and geometry of the impactor and the plate.

The method of solution of the equations of motion uses Fourier transforms in contrast to the method involving series of eigenfunctions as used by many authors. The former yields a closed form solution in terms of integrals which can be evaluated numerically. An important feature of this analysis is the determination of the transform of the contact force distribution without and with shear deformations. A somewhat involved manipulation of the integrals finally yields a very simple expression for force transform and the special cases of point load and a line distribution of load are noted.

The solutions are general enough to permit the determination of deflection and bending curvatures at an arbitrary point of the plate and not just under the point of impact. In certain special cases, exact evaluation of the integrals is possible. In particular, Boussinesq's classical solution is obtained for deflection when an orthotropic lamina/laminate is subjected to a concentrated impact load. Solutions have been obtained for bending curvatures similar to the ones as obtained by Frischbier but with a different approach. Numerical solutions for deflections and bending and twisting curvatures of a lamina/laminate are given where exact solutions are not possible. These responses have been obtained for different plates at different locations and time for a half-sine pulse, a rectangular force pulse, λ based force and for the force generated by considering shear deformations.

In actual practice, the loading is generally caused by the impact of foreign object on the plate and the history of the impact force is unknown in itself. Therefore, the determination of the impact force history is considered through relevant equations. The interaction between the plate and the impacting body is introduced through Hertz-Sveklo contact law. The effect of the contact zone is neglected. Consequently, the deflection at the point of impact due to a concentrated load is given by a simple relation for a lamina/laminate, viz. the deflection is directly proportional to the impulse imparted to the plate in case of without shear deformations. The constant of proportionality (plate parameter) depends only on the elastic constants, density and the thickness of the plate. The non-dimensionalised integral equation for impact force also depends on a single parameter called as impact parameter which is a function of plate parameter, mass and velocity of the impactor for without shear deformation case. With shear

deformations the force is determined by an iterative procedure, because in this case the central deflection is not proportional to the impulse imparted to the plate. The integral equation in the former case is solved by a simple algorithm and the curves are presented for the force history for different values of impact parameter. For the latter case the force history is obtained simultaneously with the response and plots are available for the plate and the impactor parameters chosen. After having determined the impact force history, it is now possible to determine deflection and bending and twisting curvatures and hence the bending stresses and strains. The influence of the material parameters, like the volume fraction of fibres, plate thickness, and the impactor parameters like mass, velocity and kinetic energy on impact response such as deflection, bending curvatures and stresses etc. is systematically investigated in this work. Some exact solutions for few cases were obtained and were compared with the existing literature. The singularity encountered in the expressions for bending curvatures has been overcome by considering a very small value of semi-major axis (a) of the contact ellipse in case of plates subjected to concentrated impact loads. This assumption of a very small value of a and eccentricity of the contact ellipse does not have any convergence problem on the magnitude of deflection and bending and twisting curvatures and hence the stresses induced in the plate. Value of a was varied from 0.005 m to 1×10^{-7} m and it was observed that there was no discrepancy in the response below an a value of 1×10^{-4} m.

On comparing the responses obtained with and without shear deformations, it is seen that the time of contact and the time of occurrence of maximum force are greater and the maximum force is lower in case of with shear deformations analysis for the

same impactor and plate parameters. Deflections obtained under the point of impact in case of with shear deformations are always greater than those without shear deformations at any time. Maximum curvatures, their time of occurrence and hence the stresses are greater in case of with shear deformations.

The results obtained in the present analysis are very useful to the designer. If a single impact parameter is known, the total response of the plate can be predicted and this response can be correlated to the response obtained with shear deformations. Instead of depending on the vague value of factor of safety, actual analysis can be carried out and economical structures can be designed.

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