LINEAR COMBINATIONS OF LINEAR POSITIVE OPERATORS AND GENERATING RELATIONS IN SPECIAL FUNCTIONS.

By

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CERTIFICATE

This is to certify that the thesis entitled "Linear Combinations of Linear Positive Operators and Generating Relations in Special Functions" which is being submitted by Mr. R.K.S. Rathore for the award of the degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last two and a half years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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PREFACE

The linear combinations' technique is a kind of corrective surgery on linear positive operator sequences, which brings to light the inherent, but apparently disguised, rapid approximation properties of such operator sequences. The present thesis consists of a development of this theory.

We have defined a concept of local saturation in terms of the rate of convergence of approximations for certain zero-orders by a sequence of linear positive operators. Precisely, it is this saturation that controls the order of linear combinations and the approximation by them.

Also we introduce a method of exponentials which seems to work well with many of the known positive linear operator sequences. With its help the study of convergence of operators for continuous or smoother functions becomes much simplified, at the same time giving us a better grasp of the situation. It also provides us with a very powerful method for the construction of linear positive operator sequences from certain identities in Special Functions.

The author would be amply rewarded in his efforts if the linear combinations' theory developed here makes even a slight headway towards winning the case for the linear positive operators and also if the constructive role of special functions provides a further incentive in their study.

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