LIE THEORETIC ORIGIN OF GENERATING FUNCTIONS

by

ARSHAD MAJEED

Department of Mathematics
Indian Institute of Technology, Delhi

Submitted
in fulfilment of the requirements of the degree of
DOCTOR OF PHILOSOPHY
to the
Indian Institute of Technology, Delhi

September 1980

ACKNOWLEDGEMENTS

It gives me a great pleasure to express my sincere regards and gratitude to Dr. H.L. Manocha, Head of the Department of Mathematics, Indian Institute of Technology, Delhi, for his guidance and constant encouragement throughout the preparation of this thesis.

My thanks are due to Professor O.P. Bhutani, Department of Mathematics, Indian Institute of Technology, Delhi for his encouragement and interest in my work.

I am thankful to the Director, Indian Institute of Technology, Delhi for providing me all facilities and research fellowship of the Institute.

I am very thankful to my friends, Mr. S.A.S. Kirmani and Mr. Shyam Joshi, who gave their valuable time in reading the manuscript and making detailed corrections.

Finally, I thank Mr. D.R. Joshi for his commendable work in typing the manuscript.

(ARSHAD MAJEED)

Department of Mathematics
I.I.T., Hauz Khas,
New Delhi-110016.

S Y N O P S I S

The theory of Lie groups and Lie algebra plays a fundamental role in many branches of mathematics. Research work of Lie, Engel, Cartan, Killing and Weyl have led to its recent most elegant form which is extremely useful in many applications of mathematics.

For the past three decades, mathematicians throughout the world have shown keen interest in generating functions. Various technique have been used to derive them. Representation theory of Lie groups and Lie algebra has been applied by Weisner, Vilenkin, Miller, Kalnins and Manocha to obtain important properties and generating functions of so many special functions which arise in many branches of mathematics.

In the present work theory of Lie groups and Lie algebra have been successfully used to derive new results involving special functions. Lie groups with which we come into grips, are G_4 and $SL(2)$. Details of the contents of various chapters are as follows:

<u>Chapter I</u>: This chapter deals with the main theory concerning Lie groups, Lie algebra and special functions required and used throughout the thesis. It contains two parts. Part I is addressed to Lie theory and Part II to the theory of special functions.

Chapter II: Following the work of Miller (Lie theory and special functions, New York: Academic Press 1968), Weisner [Group-theoretic origins of certain generating functions, Pacific J. Math. 5, 1033-1039) (1955)] and Kalnins, Manocha and Miller [The Lie theory of two-variable hypergeometric functions, studies in applied mathematics, Vol. 62, No.2, April 1980].

An attempt is made to obtain generating functions associated with two-variable Horn's functions H_4, H_7, H_9 etc. In the entire treatment the special linear algebra $sl(2,C)$ plays a vital role.

Chapter III: This chapter has eight sections. Section 3.1 contains generating functions for $L_n^{(\alpha)}(x)$ by α variation. The Lie algebra associated with this work is $\mathcal{G}_4$. Section 3.2 contains generating functions for $C_n^{(\alpha)}(x)$ by α variation. Lie algebra associated with this work is $sl(2,C)$. Sections 3.3, 3.4 and 3.5 contain generating functions involving $P_n^{(\alpha,\beta)}(x)$ by α variation, α-β variation, α-n variation. The Lie algebra appearing in these sections is again $sl(2,C)$. Section 3.6 and 3.7 contain generating functions for $\phi_n(c,x)$, n being a positive integer, by c-variance, c-n variation. Lie algebra associated with this work is $\mathcal{G}_4$. Finally section 3.8 contains generating functions for $\phi_n(c-2n,x)$, n being a positive integer. Lie algebra associated with this section is $sl(2,C)$.

__Chapter IV__: This chapter has two sections. In section A we define a new type of function which is a special case of $P_n^{(\alpha,\beta)}(x)$ with $\beta = -\alpha$ and we name this as anti-symmetric Jacobi function. Results obtained are parallel to those obtained by Visvanathan [Generating functions for ultra-spherical functions. Canad. J. Math. 20, 120-134 (1968)] for ultraspherical function. Section B deals with $P_n^{(\alpha-n)}(x)$, the modified ultraspherical function, in which second order differential operators have been used to find generating functions.

The Lie algebra appearing in Chapter IV is $s\ell(2,C)$.

__Chapter V__: This chapter is devoted to obtaining generating functions involving the hypergeometric function ${}_2F_1[\alpha,\beta;\gamma+n;x]$. The Lie algebra appearing in this chapter is $s\ell(2,C)$.

C O N T E N T S