

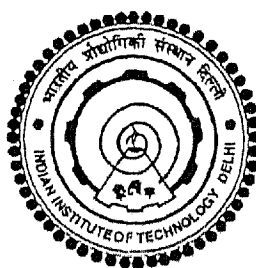
**FLOW AND DISPERSION
IN A CATHETERIZED ARTERY
- A THEORETICAL MODEL**

by

ANAMIKA SARKAR
Department of Mathematics

*Thesis submitted in fulfillment of the requirements
for the award of the degree of
Doctor of Philosophy*

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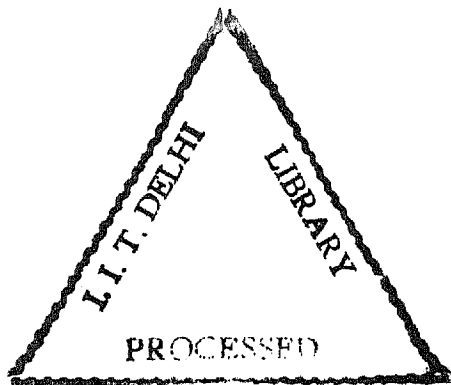


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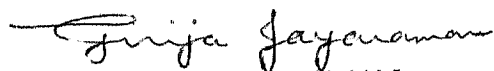
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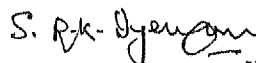
Certificate

This is to certify that the thesis entitled "FLOW AND DISPERSION IN A CATHETERIZED ARTERY – A THEORETICAL MODEL" which is being submitted by Anamika Sarkar to the INDIAN INSTITUTE OF TECHNOLOGY DELHI for the award of the degree of DOCTOR OF PHILOSOPHY in Mathematics is a record of bonafide research work carried out by her under our guidance and supervision.

The thesis has reached the standard fulfilling the requirements for the award of the degree. The results obtained in this thesis have not been submitted to any other university or institution for the award of any other degree or diploma.



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*Words
can never completely express
all that you mean to me.
Yet,
Words
are all I have
to say
that it would never
be complete
without you, Manosij.*

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A work like a PhD thesis cannot be achieved alone, but rather is a collaboration of many forces and individuals...

Foremost among them is my family, which has provided me with the strength and courage to carry on my relentless research especially at trying times.

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In the desolate world of research, friends are like an oasis...

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A
Sarkar

Anamika Sarkar

Abstract

The objective of this thesis is to understand, through mathematical models based on fluid dynamic principles, the changed dynamics of the fluid due to the *insertion of catheters* in the blood vessels. When a catheter is inserted into an artery, it will increase the impedance or frictional resistance to flow and will alter the pressure distribution. Moreover, there is an error in the measured pressure due to pressure wave reflection at the tip of the catheter. Also, catheter distortion is reported in measuring cardiac output during indicator dilution technique. Hence, there is a need to understand and quantify the changed fluid dynamic characteristics – flow, streamline patterns, pressure distribution, wall shear stress, dispersion of dyes – in the presence of a catheter.

The thesis has been broadly divided into two parts:

PART I (Chapters II-IV) deals with the modeling of the pulsatile flow in an artery, when there is a constriction or stenosis (a disease due to the focal thickening of the arterial wall resulting in occlusion of the artery).

PART II (Chapters V-VI) deals with dispersion of solutes in a catheterized artery. Generalized dispersion model is used to calculate the longitudinal diffusion coefficient.

CHAPTER I gives a brief overview of modelling in the cardiovascular system. It also includes a survey of literature which has provided the necessary motivation to study the work reported in this thesis.

CHAPTER II deals with the modelling and analysis of the changed flow pattern of pulsatile blood flow in a catheterized stenosed artery. The model takes into account the effect of the movement of the flexible catheter on the pulsatile nature of the flow. Cross-section of the artery is assumed to be

slowly varying so that the principles of lubrication theory could be used. The second order perturbation analysis for a purely oscillatory flow gives rise to steady terms – steady streaming effect. This contribution due to the steady streaming effect brings into focus the existence of a non-zero mean pressure drop in addition to the one predicted by the linear theory – a fact overlooked by previous authors. Thus, the results are intended to provide a correction to the mean pressure drop usually calculated by neglecting the non-linear inertia terms.

To understand the flow dynamics for a more realistic situation, arterial wall characteristics are taken into account in the model given in **CHAPTER III**. In this chapter, the main objective is to study the combined effect of pressure wave form and wave form of the elastic wall in the presence of a catheter. The diameter variation of the moving wall is considered small for the perturbation analysis to be valid. It is found that a correction, from the steady streaming analysis, to the mean pressure gradient predicted by linear theory depends on the diameter variation, flow rate wave forms and phase difference between the pressure gradient and the wall motion.

In **CHAPTER IV**, steady streaming analysis for the *in-vivo* experimental model of pulsatile flow in a stenosed artery in presence of a clot model is studied. The theoretical approach described in this chapter is to analyse in detail the model, especially the effect of the height of the clot model and the frequency parameters on the physiological flow parameters.

In **CHAPTER V**, using the model for dispersion in an oscillatory flow in an annulus, the dispersion of an indicator in a catheterized artery is studied. The study, based on the generalised dispersion model, brings out the development of the dispersive transport following the injection of a tracer in terms of the two effective transport coefficients viz., the convection and the dispersion coefficients.

The effect of the irreversible boundary reaction of the wall is studied in **CHAPTER VI**. The generalized dispersion model in this case, gives rise to another transport coefficient (the exchange coefficient) in addition to the convection and the dispersion coefficients, discussed in the previous chapter (Chapter V). It is found that the exchange and the convection coefficients are enhanced while the dispersion coefficient is reduced with the increase in the value of the absorption parameter.

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