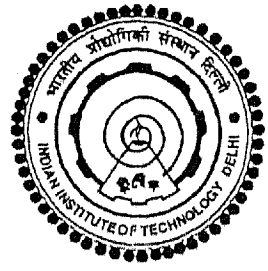
FLOW AND DISPERSION IN A CATHETERIZED ARTERY - *A THEORETICAL MODEL*

by

ANAMIKA SARKAR
Department of Mathematics

Thesis submitted in fulfillment of the requirements for the award of the degree of
Doctor of Philosophy

to the



Indian Institute of Technology Delhi
Hauz Khas, New Delhi-110 016, INDIA
November 1999

Certificate

This is to certify that the thesis entitled "FLOW AND DISPERSION IN A CATHETERIZED ARTERY – A THEORETICAL MODEL" which is being submitted by Anamika Sarkar to the INDIAN INSTITUTE OF TECHNOLOGY DELHI for the award of the degree of DOCTOR OF PHILOSOPHY in Mathematics is a record of bonafide research work carried out by her under our guidance and supervision.

The thesis has reached the standard fulfilling the requirements for the award of the degree. The results obtained in this thesis have not been submitted to any other university or institution for the award of any other degree or diploma.

Dr GIRIJA JAYARAMAN
Professor, Center for Atmospheric Sciences
Indian Institute of Technology Delhi
Hauz Khas, New Delhi - 110 016
INDIA

Dr S R K IYENGAR
Professor, Department of Mathematics
Indian Institute of Technology Delhi
Hauz Khas, New Delhi - 110 016
INDIA

Words
can never completely express
all that you mean to me.
Yet,
Words
are all I have
to say
that it would never
be complete
without you, Manosij.

Acknowledgement

I would like to thank my supervisor, Prof Girija Jayaraman, for introducing me to the interesting field of Biomechanics. I am indebted to her for her total commitment, constructive criticism, deep insights and for being a unflagging source of moral support throughout my thesis work. Furthermore, I would like to thank her for inculcating an exacting work culture besides providing me with a complete education which was not always restricted to research. Her progressive outlook towards science and need for its recognition in the international arena, was always an inspiration for me to strive harder and harder.

I would also like to thank my co-supervisor, Prof S R K Iyengar, for his intense involvement and valuable suggestions, and extending a helping hand at every juncture of the research process. I am also thankful to him for teaching me the art of efficient programming.

I am thankful to Prof B R Handa, Head of Department of Mathematics, and the authorities of Indian Institute of Technology Delhi, for providing excellent library, computing facilities and a conducive study atmosphere. I extend my sincere thanks to the staff of the Department for their cooperation throughout my research period. I would like to thank Dr Nagabhushanam and Mr Ravi Seth for understanding the need of the computing lab at odd hours and their full cooperation in this regard.

I am thankful to Prof U C Mohanty, Head of Center for Atmospheric Sciences (CAS), for permitting me to continue my PhD work during my tenure as a Project Associate in the research project (# RP069/95) of the Department of Science of Technology (DST), Government of India, under the supervision of Prof Girija Jayaraman. I am also grateful to DST for the financial assistance since the beginning of this year.

I am grateful to Council for Science and Industrial Research for providing me with the necessary travel grant for attending and presenting a paper in the World Congress of Biomechanics 1998, at Sapporo, Japan.

I would like to thank Prof B K Guha, Associate Dean (Students), IIT Delhi, for allowing me to stay in the campus as a special case during the crucial stage of thesis completion at the beginning of this year.

I am thankful to Prof J T Stuart FRS, Department of Mathematics, Imperial College of Science and Technology and Medicine, U K, for his encouraging comments in general, and in particular, for pointing out the need for specifying the phase lead of the catheter oscillations over the flow in Chapter 2 of the thesis.

I would like to express my gratitude towards Prof T J Pedley FRS, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK, for finding some precious time from his hectic schedule at Sapporo, Japan to discuss Chapter 3 of the thesis. I am grateful to him for his valuable suggestions regarding tube law, flow rate waveforms, diameter variations and axial dependence of the solutions.

A work like a PhD thesis cannot be achieved alone, but rather is a collaboration of many forces and individuals...

Foremost among them is my family, which has provided me with the strength and courage to carry on my relentless research especially at trying times.

Whatever I am today, it could not possibly be without my mother... late Purnima Sarkar. In those few years she was with me, she was able to inculcate in me the values of life and made me realise how important it is to hold my own in any situation.

My father, K B Sarkar, I would like to thank him for instilling in me the importance of education at an early age. My brother, Anupam and sister-in-law, Sonia, I would like to thank them for being their supporting selves and always being there in the hour of need.

A special thanks goes out to all members of my other family, my in-laws, for their love and affection and always believing in me in whatever I chose to do.

In the desolate world of research , friends are like an oasis...

Childhood friends are like invaluable gems in life... Dolonchapa, Swati, Debasis, Chandana who have always gave me confidence in whatever I did, and pointing out the positive points in me since my childhood, besides being my best critic.

I wish to thank my senior and friend, Ranjan, for all those interactions which made it easy for me to learn concepts of fluid mechanics, besides helping me in drawing streamlines using Surfer. I would like to thank Shalu for helping me in typing Chapter 4 of the thesis.

I consider myself very lucky to be surrounded by friends like Neela, Ragini, Promila, Namrata, Sankalpa, and Arun, for being an equal partner, seeing me through thick and thin, and shouldering my travails as if it were their own.

Persons become special who see us through difficult times. I am grateful to Arindam for always going that extra mile for keeping my morale high and boosting my confidence, especially during the final stages of my thesis work. He cannot be thanked in mere words.

This acknowledgement cannot end without the mention of my best friend and husband, Manosij, who not only provided me with incessant support and encouragement, but also made my research and thesis work into an enjoyable experience.

A Sarkar

Anamika Sarkar

Abstract

The objective of this thesis is to understand, through mathematical models based on fluid dynamic principles, the changed dynamics of the fluid due to the *insertion of catheters* in the blood vessels. When a catheter is inserted into an artery, it will increase the impedance or frictional resistance to flow and will alter the pressure distribution. Moreover, there is an error in the measured pressure due to pressure wave reflection at the tip of the catheter. Also, catheter distortion is reported in measuring cardiac output during indicator dilution technique. Hence, there is a need to understand and quantify the changed fluid dynamic characteristics – flow, streamline patterns, pressure distribution, wall shear stress, dispersion of dyes – in the presence of a catheter.

The thesis has been broadly divided into two parts:

PART I (Chapters II-IV) deals with the modeling of the pulsatile flow in an artery, when there is a constriction or stenosis (a disease due to the focal thickening of the arterial wall resulting in occlusion of the artery).

PART II (Chapters V-VI) deals with dispersion of solutes in a catheterized artery. Generalized dispersion model is used to calculate the longitudinal diffusion coefficient.

CHAPTER I gives a brief overview of modelling in the cardiovascular system. It also includes a survey of literature which has provided the necessary motivation to study the work reported in this thesis.

CHAPTER II deals with the modelling and analysis of the changed flow pattern of pulsatile blood flow in a catheterized stenosed artery. The model takes into account the effect of the movement of the flexible catheter on the pulsatile nature of the flow. Cross-section of the artery is assumed to be

slowly varying so that the principles of lubrication theory could be used. The second order perturbation analysis for a purely oscillatory flow gives rise to steady terms – steady streaming effect . This contribution due to the steady streaming effect brings into focus the existence of a non-zero mean pressure drop in addition to the one predicted by the linear theory – a fact over looked by previous authors. Thus, the results are intended to provide a correction to the mean pressure drop usually calculated by neglecting the non-linear inertia terms.

To understand the flow dynamics for a more realistic situation, arterial wall characteristics are taken into account in the model given in **CHAPTER III.** In this chapter, the main objective is to study the combined effect of pressure wave form and wave form of the elastic wall in the presence of a catheter. The diameter variation of the moving wall is considered small for the perturbation analysis to be valid. It is found that a correction, from the steady streaming analysis, to the mean pressure gradient predicted by linear theory depends on the diameter variation, flow rate wave forms and phase difference between the pressure gradient and the wall motion.

In **CHAPTER IV,** steady streaming analysis for the *in-vitro* experimental model of pulsatile flow in a stenosed artery in presence of a clot model is studied. The theoretical approach described in this chapter is to analyse in detail the model, especially the effect of the height of the clot model and the frequency parameters on the physiological flow parameters.

In **CHAPTER V,** using the model for dispersion in an oscillatory flow in an annulus, the dispersion of an indicator in a catheterized artery is studied. The study, based on the generalised dispersion model, brings out the development of the dispersive transport following the injection of a tracer in terms of the two effective transport coefficients viz., the convection and the dispersion coefficients.

The effect of the irreversible boundary reaction of the wall is studied in **CHAPTER VI.** The generalized dispersion model in this case, gives rise to another transport coefficient (the exchange coefficient) in addition to the convection and the dispersion coefficients, discussed in the previous chapter (Chapter V). It is found that the exchange and the convection coefficients are enhanced while the dispersion coefficient is reduced with the increase in the value of the absorption parameter.

Table of Contents