

FINITE ELEMENT ANALYSIS OF  
LAMINATED COMPOSITE PLATE  
AND SHELL STRUCTURES

by

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CERTIFICATE

This is to certify that the thesis entitled "Finite Element Analysis of Laminated Composite Plate and Shell Structure" being submitted by Mr. Suresh Chandra Panda to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Civil Engineering, is a record of bonafide research work carried out by him. He has worked under my guidance and has fulfilled the requirements for the submission of this thesis, which has reached the requisite standard.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any Degree or Diploma.

  
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ABSTRACT

Laminated fibre reinforced composites are increasingly used as structural components particularly in weight sensitive aircraft and aerospace structures. Theoretical analyses of these type of structures are limited to relatively simple geometry, load and boundary conditions. If these conditions are more complex, the analysis becomes increasingly tedious and even impossible.

In this thesis finite element formulations are presented for the analyses of laminated composite structures. The formulations include arbitrary number of bonded orthotropic layers, each of which can orient in arbitrary direction. Transverse shear deformation - an essential feature of all composite construction - is included by allowing the straight normal to rotate but assumed to remain straight. Stress strain relation is derived from three-dimensional elasticity approach. The finite elements adopted are based on isoparametric concept. Separate formulations are presented for the analyses of laminated plate, axisymmetric shell and doubly curved shell structures.

Several problems have been solved to demonstrate the validity of the formulations and their application to practical structures. Finite element results are compared with exact theoretical solution wherever it is available. Three-layer graphite-epoxy plate solution is also compared with another

finite element results. A liquid-propellant rocket nozzle having complex wall construction is analysed using axisymmetric shell element and the results are compared with a known finite difference solution. Using the same element, results are presented for stress distribution in human left ventricular layered wall. The doubly curved shell element is used for the design analysis of a below-knee symc prosthesis with boron-epoxy laminate wall construction.

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