FINITE DEFORMATION MODELING AND ANALYSIS OF STRUCTURAL INSTABILITIES IN THIN MAGNETOELASTIC SHEETS

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Finite Deformation Modeling and Analysis of Structural Instabilities in Thin Magnetoelastic Sheets

by

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Dedicated to
my parents

Certificate

This is to certify that the thesis entitled **'Finite Deformation Modeling and Analysis of Structural Instabilities in Thin Magnetoelastic Sheets'** submitted by **Ms. Awantika Mishra** to the **Indian Institute of Technology Delhi** for the award of the degree of **Doctor of Philosophy** embodies original research work done by her under my supervision. In my opinion, the thesis work meets the requisite standards and the candidate is worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the institute. The contents of this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Date:

Place: New Delhi, India

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It is possible that I may have inadvertently omitted certain individuals of significant importance in this acknowledgment, an oversight stemming from my memory lapse. Nevertheless, it is essential to note that my acknowledgment extends beyond the scope of these pages.

Awantika Mishra

Abstract

This dissertation deals with the nonlinear finite deformation modelling and analysis of magnetorheological elastomers (MREs) and associated structural instabilities.

MREs are smart materials that undergo changes in their magnetic and rheological properties under the application of an external magnetic field. This distinctive behavior enables these materials to be utilized in a wide range of engineering and biomedical applications. In this work, a two dimensional model is developed using variational formulation to study the beha viour of thin magnetoelastic sheets for different geometries and loading conditions.

The first part of the dissertation deals with the computational analysis of snap through/limit point instability in magnetoelastic membrane for circular and cylindrical geometry. An $O(h)$ membrane theory is formulated and the resulting nonlinear system of ordinary differential equations are solved using a finite difference based boundary value problem solver. The theory is subsequently applied to a magnetoelastic membrane of circular geometry under coupled loading conditions and the formulation is further extended to study an electro-magneto-mechanical membrane of cylindrical geometry. The results obtained are first verified with the existing literature for each geometry. The deformation and instabilities in the membrane are studied under axisymmetric loading and applied transverse fields while incorporating material nonlinearity. For the circular geometry, effect of Maxwell stresses and Pre-stretch on the deformation of the membrane is studied. The solver is prone to convergence issues due to high sensitivity to initial guess, particularly at higher loading conditions. A computational scheme to address these issues is devised, and solutions are obtained for different material properties. Finally, parametric analysis is performed by varying magnetic field strength and pressure inputs to study the nature of limit point instability for different loading conditions. For the cylindrical geometry, additional coupling effect of electric field is incorporated in the formulation. The extended model is subsequently used to analyze mechanical and electrical limit-point instabilities and the effect of external fields on the onset of these instabilities. It is observed that the onset of mechanical limit-point instability is dependent on the material properties, geometrical parameters, and applied electromagnetic fields. Magnetic field-induced instability results in an initial dip in the pressure versus stretch plots, subsequently converging to purely hyperelastic behavior at larger stretches. On the other hand, applying an electric field results in an early onset of limit-point instability (i.e., at smaller stretches) compared to the hyperelastic case. In summary, our membrane model describes the interactions between electromagnetic fields. The electric, magnetic and elastic limit points observed in these membranes under different set of loading conditions can be used to improve the design of soft actuator and energy-harvesting devices.

The second part of the dissertation deals with the development of $O(h^3)$ finite-deformation plate theory for a thin magnetoelastic sheet starting from a 3D variational form. This theory is used to model wrinkling phenomena under applied traction and an external magnetic field. The resulting 2D potential energy is specialized to an isotropic, incompressible material, and the expressions for material constants are obtained as a function of the applied magnetic field. Strong form of the resulting 2D governing equations are obtained and the resulting governing equations are used to analyze wrinkling in a stretched rectangular sheet placed in a uniform magnetic field. Approximate analytical solutions are obtained by linearizing the 2D plate equation for small slopes. The effect of the magnetic field and the applied stretch on the onset of wrinkling and amplitude of wrinkles is studied. It was observed that the amplitude of wrinkles decrease and the stiffness of the material increases with the magnetic field.

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List of Symbols

κ_r	Reference configuration of the membrane, see equation (3.16), page 24
Ω_r	Mid-surface of the undeformed membrane, see equation (5.111), page 118
$\partial\kappa$	Surface boundary of deformed configuration, see equation (3.19), page 25
$\partial\kappa_r$	surface boundary of the reference configuration, see equation (3.17), page 24
ϵ_0	electrical permittivity of the free space, see equation (4.5), page 53
μ	Absolute permeability of the material, see equation (3.47), page 32
μ_0	Absolute magnetic permeability of the free space, see equation (3.18), page 25
ρ_r	Density of the material in reference configuration, see equation (3.44), page 31
σ_F	Surface charge density per unit reference area, see equation (B.9), page 142
σ_f	Surface charge density per unit deformed area, see equation (B.7), page 142
$\mathbf{G}$	Shear modulus of the material, see equation (3.47), page 32
W	Free energy of the material per unit reference volume, see equation (3.6), page 21
$\mathbf{b}$	Spatial magnetic flux density, see equation (3.19), page 25
$\mathbf{C}$	Cauchy strain tensor, see equation (3.47), page 32
$\mathbf{D}$	Lagrangian electric displacement vector, see equation (4.1), page 52
$\mathbf{D}$	Lagrangian electric displacement vector, see equation (4.5), page 53
$\mathbf{D}$	Lagrangian electric displacement vector, see equation (B.7), page 142
$\mathbf{E}$	Electric field strength in the reference configuration, see equation (4.2), page 52
$\mathbf{E}$	Lagrangian electric field strength, see equation (4.1), page 52
$\mathbf{F}$	Deformation gradient, see equation (3.3), page 21

$\mathbf{H}$	Magnetic field strength in the reference configuration, see equation (3.12), page 23
$\mathbf{h}$	Spatial magnetic field intensity, see equation (3.19), page 25
$\mathbf{P}$	Piola Kircoff's stress tensor, see equation (3.6), page 21
$\mathbf{R}$	Rotation tensor, see equation (3.4), page 21
$\mathbf{S}$	Second Piola Kircoff's stress tensor, see equation (3.6), page 21
$\mathbf{t}_a$	Applied traction per unit deformed area, see equation (3.17), page 24
$\mathbf{T}_m$	Maxwell stress tensor due to external applied field in the free space, see equation (3.17), page 24
$\mathbf{U}$	Right stretch tensor, see equation (3.4), page 21
$\mathbf{V}$	Left stretch tensor, see equation (3.4), page 21
$\mathbf{B}$	Referential magnetic flux density, see equation (3.16), page 24
β	angle between the normal vectors of deformed and undeformed membrane , see equation (3.57), page 35
$\chi(\mathbf{X})$	Deformation map, see equation (3.2), page 20
J	Determinant of deformation tensor, see equation (A.5), page 137
$\mathbf{x}$	Position of arbitrary point in the deformed configuration, see equation (3.2), page 20
λ_i	Principal stretches in the principal directions, see equation (3.5), page 21
ϕ	magnetostatic potential, see equation (3.12), page 23
φ_e	electrostatic potential, see equation (4.2), page 52
$\mathbf{d}(\mathbf{u}), \mathbf{g}(\mathbf{u})$	director vectors, see equation (3.9), page 22
$\mathbf{r}(\mathbf{u})$	Projection of the arbitrary point on the deformed mid-surface, see equation (3.7), page 21
$\mathbf{u}$	Projection of arbitrary point on the undeformed mid-surface, see equation (3.1), page 20

$\mathbf{X}$	Position of arbitrary point in the reference configuration, see equation (3.1), page 20
η	through thickness coordinate in the reference configuration, see equation (3.1), page 20
h	Thickness of the membrane, see equation (A.6), page 137
$\mathbf{N}$	Unit normal vector on reference surface boundary, see equation (3.17), page 24
$\mathbf{n}$	Unit normal vector on deformed surface boundary, see equation (3.17), page 24
$\mathbf{N}_3$	normal to the undeformed mid-surface of the membrane, see equation (3.1), page 20
$\mathbf{N}_\alpha$	In-plane coordinates in the undeformed configuration, see equation (3.11), page 22
$\mathbf{n}_\alpha$	In-plane coordinates in the deformed configuration, see equation (3.11), page 22
$\mathbf{N}_i$	Orthonormal basis in undeformed configuration, see equation (3.5), page 21
$\mathbf{n}_i$	Orthonormal basis in deformed configuration, see equation (3.5), page 21
∇	2-D gradient operator with respect to the reference coordinates, see equation (3.8), page 22
Curl	3-D curl operator corresponding to reference configuration, see equation (3.16), page 24
Div	3-D divergence operator corresponding to reference configuration, see equation (3.16), page 24