

DISTRIBUTED CONTROL FOR SYNCHRONIZATION IN DESCRIPTOR MULTI-AGENT SYSTEMS

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DEPARTMENT OF ELECTRICAL ENGINEERING
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by

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Submitted

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to the



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CERTIFICATE

This is to certify that the thesis entitled **Distributed Control for Synchronization in Descriptor Multi-Agent Systems** submitted by **Meera Patel** to the Indian Institute of Technology Delhi for the award of the Degree of **Doctor of Philosophy**, is a record of the bona fide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Prof. Subashish Datta
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(Supervisor)

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Meera Patel

ABSTRACT

A multi-agent system comprises a group of dynamical systems, with an individual system being referred to as an *agent*. These agents work cooperatively via appropriate communication links to achieve some specific collective objectives such as consensus, formation, and synchronization. When the dynamics of an agent involve differential and algebraic equations, it can be represented as a descriptor system (DS): $E\dot{x}(t) = Ax(t) + Bu(t)$, where the matrix E is a *singular* matrix. Many physical systems with *state constraints* (represented using algebraic equations) are often suitably modeled as descriptor systems. The examples of such systems are robotics, power networks, biological systems, and cyber-physical systems. Some of the characteristics that make DSs different from ordinary state-space systems are: the state response of a DS may not exist if it is not *regular*, and could be *impulsive* for non-smoothness control input and/or inconsistent initial conditions. The presence of impulse in the system response may damage or saturate the physical components, and hence, it should be eliminated. A network of such descriptor systems is referred to as a *descriptor multi-agent system* (DMAS). In a multi-agent setting, the distributed feedback control architecture is preferred in comparison to centralized feedback control due to several advantages, such as a reduction in the number of communication channels and avoiding single-point failure. In a distributed feedback control, each agent uses the state or output information only from the neighboring agents. In this regard, this thesis considers the design of a distributed control architecture for a DMAS to address the leader-follower synchronization problem and *eliminate the impulsive response* in the closed-loop network. In *leader-follower synchronization*, a leader agent generates a desired reference trajectory, and a control law is designed to ensure that the states of the follower agents synchronize with the states of the leader agent.

In particular, the initial work of this thesis focuses on the asymptotic convergence of leader-follower agents while also addressing the elimination of impulsive responses in the closed-loop system. To achieve this control objective, a distributed static state feedback control law is proposed, where the closed-loop DMAS is made impulse-free through a feedback gain matrix. Furthermore, the system is transformed into a set of decoupled descriptor systems using the property of the network graph Laplacian matrix. Then, by performing appropriate decomposition of the transformed descriptor systems, the synchronization objective is achieved by stabilizing a set of ordinary state space systems through solving an Algebraic Riccati Equation. Next, we consider another problem where the states of follower agents synchronize with the states of leader agents satisfying a pre-specified *minimum decay rate*, which helps to regulate

the synchronization speed. Moreover, the impulse-free closed-loop response is also guaranteed. By performing a sequence of system transformations through orthogonal matrices and using a *parametric robust control* framework, the synchronization problem is addressed. The corresponding gain parameters are computed by solving a linear matrix constraint problem and using the largest eigenvalue of the network Laplacian matrix. These control schemes are developed under the assumption that all states are measurable, which is then used to design a static state feedback control. Nevertheless, the states can not always be measurable in practice due to the lack of sensors or cost considerations. Therefore, the subsequent work of the thesis focuses on designing a *local observer-distributed control* scheme for DMAS, allowing the agents interaction to be cooperative and antagonistic. For this, a *reduced-order observer* of order $n_0 = \text{rank}(E)$ is proposed to estimate the states of all the agents in the network. The proposed control architecture comprises an *output-based* local control law to eliminate impulsive responses and an *estimated state-based distributed feedback law* to achieve the synchronization objective. The feedback control algorithm provides flexibility to design controller and observer gain matrices independently, and hence, the standard *separation principle* holds. Moreover, considering only cooperative interactions among the agents, the proposed observer-based control law is extended to achieve synchronization for an uncertain network graph. The associated feedback gain parameters are designed using system transformation through orthogonal matrices only. Therefore, the developed feedback control algorithm is numerically efficient. The effectiveness of developed algorithms is demonstrated with numerical examples and practical example such as microgrids.

Keywords: Descriptor Multi-Agent System, Impulse Elimination, Leader-Follower Synchronization, Robust Control Approach, Constant Reference Tracking, Observer-based Controller, Linear Matrix Inequality.

सार

मल्टी-एजेंट सिस्टम में गतिशील सिस्टमस का एक समूह शामिल होता है, जिसमें प्रत्येक सिस्टमस को एजेंट के रूप में संदर्भित किया जाता है। ये एजेंट कुछ विशिष्ट सामूहिक उद्देश्यों जैसे आम सहमति, गठन और सिंक्रनाइज़ेशन को प्राप्त करने के लिए उपयुक्त संचार लिंक के माध्यम से सहकारी रूप से काम करते हैं। जब किसी एजेंट की गतिकी में अवकल समीकरण और बीजगणितीय समीकरण शामिल होते हैं, तो इसे डिस्क्रेट सिस्टम $E\dot{x}(t) = Ax(t) + Bu(t)$ के रूप में दर्शाया जा सकता है, जहां मैट्रिक्स E एक विशेष मैट्रिक्स है। बहुत से भौतिक सिस्टम जिनमें स्थिति प्रतिबंध होते हैं (जिन्हें बीजगणितीय समीकरणों के माध्यम से व्यक्त किया जाता है), अक्सर उपयुक्त रूप से डिस्क्रेट सिस्टम के रूप में मॉडल किए जाते हैं। ऐसे सिस्टमों के उदाहरण में रोबोटिक्स, पावर नेटवर्क्स, जैविक सिस्टम, और साइबर-फिजिकल सिस्टम शामिल हैं। कुछ विशेषताएँ जो डिस्क्रेट सिस्टम को सामान्य स्टेट-स्पेस सिस्टमों से अलग बनाती हैं, वे हैं: यदि डिस्क्रेट सिस्टम सामान्य नहीं है, तो इसका स्टेट प्रतिक्रिया मौजूद नहीं हो सकती, और यदि नियंत्रण इनपुट असमाधान हो या प्रारंभिक शर्तें असंगत हों, तो प्रतिक्रिया में आवेग आ सकता है। सिस्टम प्रतिक्रिया में आवेग की उपस्थिति भौतिक घटकों को नुकसान पहुँचा सकती है या उन्हें संतृप्त कर सकती है, और इसलिए इसे समाप्त किया जाना चाहिए। ऐसे डिस्क्रेट सिस्टमों के एक नेटवर्क को डिस्क्रेट मल्टी-एजेंट सिस्टम कहा जाता है। मल्टी-एजेंट सेटिंग में, वितरित फीडबैक नियंत्रण आर्किटेक्चर को केंद्रीकृत फीडबैक नियंत्रण की तुलना में कई लाभों के कारण प्राथमिकता दी जाती है, जैसे कि संचार चैनलों की संख्या में कमी और एकल बिंदु विफलता से बचाव। वितरित फीडबैक नियंत्रण में, प्रत्येक एजेंट केवल पड़ोसी एजेंटों से स्थिति या आउटपुट जानकारी का उपयोग करता है। इस संदर्भ में, इस थिसिस में डिस्क्रेट मल्टी-एजेंट सिस्टम के लिए एक वितरित नियंत्रण आर्किटेक्चर के डिज़ाइन पर विचार किया गया है, जिसका उद्देश्य लीडर-फॉलोअर सिंक्रनाइज़ेशन समस्या को संबोधित करना और बंद-लूप नेटवर्क में आवेगी प्रतिक्रिया को समाप्त करना है। लीडर-फॉलोअर सिंक्रनाइज़ेशन में, एक लीडर एजेंट एक वांछित संदर्भ पथ उत्पन्न करता है, और एक नियंत्रण कानून डिज़ाइन किया जाता है ताकि फॉलोअर एजेंटों की स्थिति लीडर एजेंट की स्थिति के साथ सिंक्रनाइज़ हो सके।

विशेष रूप से, इस थिसिस का प्रारंभिक कार्य लीडर-फॉलोअर एजेंटों की असम्टोटिक संगति पर ध्यान केंद्रित करता है, साथ ही बंद-लूप सिस्टम में आवेगी प्रतिक्रियाओं के उन्मूलन को भी संबोधित करता है। इस नियंत्रण लक्ष्य को प्राप्त करने के लिए, एक वितरित स्थैतिक स्थिति फीडबैक नियंत्रण कानून प्रस्तावित किया गया है, जिसमें बंद-लूप डिस्क्रेट मल्टी-एजेंट सिस्टम को एक फीडबैक गेन के माध्यम से आवेग-मुक्त बनाया जाता है। इसके अतिरिक्त, सिस्टम को नेटवर्क ग्राफ लैप्लासियन मैट्रिक्स की विशेषता का उपयोग करके एक सेट से वियुग्मित डिस्क्रेट सिस्टमों में परिवर्तित किया जाता है। इसके बाद, परिवर्तित डिस्क्रेट सिस्टमों

की उपयुक्त विघटन के द्वारा, सिंक्रनाइज़ेशन लक्ष्य को प्राप्त किया जाता है। इसमें एक सामान्य स्टेट-स्पेस सिस्टमों के सेट को स्थिर करके एक बीजगणितीय रिक्काटी समीकरण को हल किया जाता है। इसके बाद, हम एक अन्य समस्या पर विचार करते हैं जहां फॉलोअर एजेंटों की स्थिति लीडर एजेंटों की स्थितियों के साथ सिंक्रनाइज़ होती है, जो एक पूर्व-निर्धारित न्यूनतम क्षय दर को पूरा करती है, जो सिंक्रनाइज़ेशन की गति को नियंत्रित करने में मदद करती है। इसके अतिरिक्त, आवेग-मुक्त बंद-लूप प्रतिक्रिया की भी गारंटी दी जाती है। ऑर्थोगोनल मैट्रिसेस के माध्यम से सिस्टम परिवर्तनों का अनुक्रम करके और पैरामीट्रिक रॉबस्ट कंट्रोल ढांचे का उपयोग करके, सिंक्रनाइज़ेशन समस्या को संबोधित किया जाता है। संबंधित गेन पैरामीटर्स को एक लीनियर मैट्रिक्स कंस्ट्रेंट समस्या को हल करके और नेटवर्क लैप्लासियन मैट्रिक्स के सबसे बड़े ईगेनवैल्यू का उपयोग करके गणना किया जाता है। ये नियंत्रण योजनाएँ इस धारणा के तहत विकसित किया गया है कि सभी स्थितियाँ मापी जा सकती हैं, जिसे फिर स्थैतिक स्थिति फीडबैक नियंत्रण डिजाइन करने के लिए उपयोग किया जाता है। फिर भी, प्रैक्टिकल में स्थितियाँ हमेशा मापी जा सकने योग्य नहीं होतीं, क्योंकि या तो सेंसर की कमी होती है या लागत की चिंताओं के कारण। इसलिए, इस थिसिस का अगला कार्य डेस्क्रिप्टर मल्टी-एजेंट सिस्टम के लिए एक 'स्थानीय पर्यवेक्षक-वितरित नियंत्रण योजना' डिजाइन पर केंद्रित है, जिससे एजेंटों की इंटरैक्शन सहकारी और विरोधी हो सकती है। इसके लिए, नेटवर्क में सभी एजेंटों की स्थितियों का अनुमान लगाने के लिए ऑर्डर $n_0 = \text{rank}(E)$ का एक घटित-क्रम पर्यवेक्षक का प्रस्ताव किया गया है। प्रस्तावित नियंत्रण आर्किटेक्चर में एक आउटपुट-आधारित स्थानीय नियंत्रण कानून शामिल है, जो आवेगी प्रतिक्रियाओं को समाप्त करने के लिए है, और एक अनुमानित स्थिति-आधारित वितरित फीडबैक कानून शामिल है, जो सिंक्रनाइज़ेशन लक्ष्य को प्राप्त करने के लिए है। प्रतिक्रिया नियंत्रण एल्गोरिथ्म स्वतंत्र रूप से नियंत्रक और पर्यवेक्षक लाभ मैट्रिक्स को डिजाइन करने के लिए लचीलापन प्रदान करता है, और इसलिए, मानक पृथक्करण सिद्धांत धारण करता है। इसके अलावा, एजेंटों के बीच केवल सहकारी बातचीत पर विचार करते हुए, प्रस्तावित पर्यवेक्षक-आधारित नियंत्रण कानून को अनिश्चित नेटवर्क ग्राफ के लिए सिंक्रनाइज़ेशन प्राप्त करने के लिए बढ़ाया गया है। संबंधित फीडबैक गेन पैरामीटर्स को केवल ऑर्थोगोनल मैट्रिक्स के माध्यम से सिस्टम ट्रांसफॉर्मेशन का उपयोग करके डिज़ाइन किया गया है। इसलिए, विकसित किया गया फीडबैक नियंत्रण एल्गोरिदम गणनात्मक रूप से कुशल है। विकसित किए गए एल्गोरिदम की प्रभावशीलता संख्यात्मक उदाहरणों और व्यावहारिक उदाहरणों जैसे कि माइक्रोग्रिड्स के साथ प्रदर्शित की गई है।

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List of Abbreviations

DS	Descriptor System
MAS	Multi-Agent System
DMAS	Descriptor Multi-Agent System
LMI	Linear Matrix Inequality
LMC	Linear Matrix Constraint
LTI	Linear Time-Invariant
ARE	Algebraic Riccati Equation
SVD	Singular Value Decomposition

List of Symbols

s	Scalar s
$\otimes$	Kronecker Product
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	n -dimensional Euclidean space
$\mathbb{R}^{m \times n}$	Space of $m \times n$ real matrices
$\mathbb{C}$	Set of complex numbers
$\mathbb{C}^-$	Complex left half of the plane
κ	Index of a Nilpotent matrix N
$\mathbb{C}_p^{\kappa-1}$	$\kappa - 1$ times piecewise continuously differentiable function set
$\exists$	there exists
$\forall$	for all
$\mathbf{P} \succ \mathbf{0}$	Positive definite matrix $\mathbf{P}$
$\mathbf{P} \succeq \mathbf{0}$	Positive semidefinite matrix $\mathbf{P}$
$\mathbf{P} \prec \mathbf{0}$	Negative definite matrix $\mathbf{P}$
$\mathbf{P} \preceq \mathbf{0}$	Negative semidefinite matrix $\mathbf{P}$
$\sigma_{\max}(\cdot)$	Maximum singular value of the matrix
$\lambda_{\min}(\mathbf{P})$	Minimum eigenvalue of the matrix $\mathbf{P}$
$\lambda_{\max}(\mathbf{P})$	Maximum eigenvalue of the matrix $\mathbf{P}$
$ \mathcal{N} $	Cardinality of the set $\mathcal{N}$
$\mathbf{I}_n$	Identity matrix with dimension $n \times n$
$\text{diag}\{\cdot\}$	A diagonal matrix
$\deg\{\cdot\}$	Degree of a polynomial
$\det\{\cdot\}$	Determinant of a matrix
$\text{eig}\{\cdot\}$	Eigenvalues of a matrix
$\text{fnp}\{\cdot, \cdot\}$	set of all finite poles
$\rho\{\cdot\}$	Spectral radius of a matrix