

**FRACTURE PROPAGATION IN WEAK SNOW
LAYERS AND FAILURE OF OVERLYING
SNOW SLAB**

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**DEPARTMENT OF APPLIED MECHANICS
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by

AGRAJ UPADHYAY

DEPARTMENT OF APPLIED MECHANICS

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

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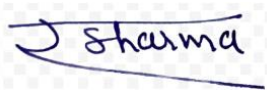
INDIAN INSTITUTE OF TECHNOLOGY DELHI

MAY 2023

Dedicated to my beloved parents and family

Certificate

This is to certify that the thesis entitled “**FRACTURE PROPAGATION IN WEAK SNOW LAYERS AND FAILURE OF OVERLYING SNOW SLAB**” being submitted by Mr. **Agraj Upadhyay** is the report of bonafide research work carried out by him under our supervision. This thesis has been prepared in conformity with the rules and regulations of the INDIAN INSTITUTE OF TECHNOLOGY, DELHI. I further certify that the thesis has attained a standard required for the award of DOCTOR OF PHILOSOPHY degree of the institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.



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Agraj Upadhyay

Abstract

Fracture propagation in the weak snow layer and subsequent failure of the overlying slab, two essential processes of the slab avalanche formation, are studied in the present work. First, mechanical behavior of natural snow is characterized through uniaxial mechanical tests and density dependent power law expressions for snow properties are obtained. For simulation studies, a continuum isotropic elastic-plastic damaging material model with Drucker-Prager yield criterion and fracture energy-based softening is considered for the snow layers and model parameters are derived from the experimental data. In addition, a discrete ice column model for the surface hoar like weak snow layer is also conceptualized and used in numerical propagation saw tests (PST). The effective mechanical behaviour of ice column model is also determined through numerical tests and is found close to that of low-density snow.

Critical crack length (CCL) and fractured slab length (FSL) are studied in the standard field PST as well as in improvised saw test in natural snow cover. Improvised saw tests in natural snow cover suggest much higher critical crack size in comparison to the CCL in field PSTs. In numerical PST study natural snow cover is idealized as a three-layer snow cover with the ice column weak layer sandwiched between two homogeneous layers. CCL and fracture propagation speed (FPS) are determined from these simulations and the influence of slope angle, weak layer and overlying slab properties on CCL and FPS is presented. An analysis of the energy released and its distribution during dynamic fracture propagation in the weak snow layer is also discussed. A low-density continuum model of snow is also used as a weak layer in PST simulations and the results obtained are comparable to ice column weak layer model in terms of trends of variation. A study on crack tip stresses in the weak snow layer is also presented. Crack tip stresses are estimated through static FE simulations in a slab-weak layer model with a pre-existing crack. The variation of crack tip stresses with slope angle is discussed and compared with results in the literature. CCL in PST is also estimated using energy

approach. Effect of slope angle, slab properties and weak layer fracture energy on CCL is discussed and results are also compared with previous models. No major influence of the deformability of the weak snow layer on CCL variation with slope angle is observed though a lower CCL is obtained for compliant weak layer. A comparative study on critical crack size in idealized 3 D terrain and in field PST is also presented.

Above numerical model of PST with ice column weak layer is also used to study the failure of snow slab. The PST simulation results on the nature of slab failure, influence of top slab density, thickness, slope angles etc. on FSL are discussed and compared with field PST results. An analysis of stresses in the snow slab and their role on the snow slab failure is also discussed. The above continuum model of snow is also used to simulate the field PSTs and a comparison of simulation and field PST results is presented. The application of above continuum model of snow is also extended to simulations in idealized 2D and 3D terrains. FPS in weak snow layer and its variation with fracture propagation distance in upslope, downslope and across the slope (in 3 D only) directions is presented. Nature of snow slab failure in upslope and downslope directions and influence of slope angle, slab density and thickness on FSL in 2D terrain are discussed. In 3D terrain, failure in the snow slab is found to initiate in the upslope direction followed by downslope and flank failure.

Overall, the above studies provide a reasonably good insight into the fracture propagation process in the weak snow layer and failure of overlying slab. However, due the computational limitations, only limited understanding on snow slab failure in 3D terrain is generated.

सार

स्लैब अवधाव बनने की दो आवश्यक प्रक्रियाओं, कमजोर हिमपरत में फ्रैक्चर का प्रसार तथा तत्पश्चात उसके ऊपर स्थित हिमपरत के टूटने की प्रक्रिया, का अध्ययन वर्तमान कार्य में किया गया है। सर्वप्रथम प्राकृतिक हिम के यांत्रिक व्यवहार को एकअक्षीय यांत्रिक परीक्षण द्वारा प्राप्त किया गया है तथा हिम के गुणों के लिए घनत्व आधारित घातीय समीकरण प्राप्त किए गये हैं। सिमुलेशन अध्ययन के लिए हिम परतों के लिए ड्रकर-प्रेगर यील्ड मानदंड तथा फ्रैक्चर ऊर्जा-आधारित सॉफ्टनिंग वाला एक सातत्य, समदैशिक, इलास्टिक-प्लास्टिक-डैमेजिंग पदार्थ मॉडल का चयन किया गया है तथा मॉडल पैरामीटर प्रयोगात्मक आंकड़ों से प्राप्त किए गए हैं। इसके अतिरिक्त, सरफेस होर नामक कमजोर हिम परत के लिए एक आइस कॉलम वाले असतत मॉडल की भी अवधारणा की गई है जिसे नुमेरिकल प्रोपेगेशन सॉ टेस्ट (पीएसटी) में प्रयोग किया गया है। आइस कॉलम मॉडल का प्रभावी यांत्रिक व्यवहार भी संख्यात्मक कम्प्यूटरीकृत परीक्षणों के माध्यम से प्राप्त किया गया है जिसे कम घनत्व वाले हिम के निकट पाया गया।

मानक फील्ड पीएसटी के साथ-साथ प्राकृतिक हिम परतों में इम्प्रोवाइज्ड सॉ टेस्ट के माध्यम से भी क्रिटिकल क्रैक लेंथ (सीसीएल) और फ्रैक्चर्ड स्लैब लेंथ (एफएसएल) का अध्ययन किया गया है। प्राकृतिक हिम परतों में किए गए इम्प्रोवाइज्ड सॉ टेस्ट ने फील्ड पीएसटी में प्राप्त सीसीएल की तुलना में कहीं अधिक क्रिटिकल क्रैक साइज़ की जानकारी दी। संख्यात्मक कम्प्यूटरीकृत पीएसटी अध्ययन के लिए प्राकृतिक हिम परतों के आवरण को दो सजातीय हिम परतों के बीच में आइस कॉलम वाली कमजोर हिम परत के साथ तीन हिम परत वाले हिम आवरण के रूप में आदर्शित किया गया। इन सिमुलेशनों से सीसीएल और फ्रैक्चर प्रसार गति (एफपीएस) प्राप्त की गयी तथा सीसीएल और एफपीएस पर ढलान कोण, कमजोर हिम परत और ऊपरी हिम परत (स्लैब) के गुणों के प्रभाव को प्रस्तुत किया गया है। कमजोर हिम परत में गतिशील फ्रैक्चर प्रसार के दौरान प्राप्त ऊर्जा और उसके वितरण के विश्लेषण पर भी चर्चा की गई है। पीएसटी सिमुलेशन में एक कमजोर हिम परत के रूप में हिम के एक कम घनत्व वाले सातत्य मॉडल का भी प्रयोग किया गया और प्राप्त परिणाम (रूझान) आइस कॉलम वाले कमजोर परत मॉडल के समान पाये गए हैं। कमजोर हिम परत में क्रैक टिप स्ट्रेस पर भी एक अध्ययन प्रस्तुत किया गया है। पहले से मौजूद क्रैक वाले स्लैब-कमजोर हिम परत मॉडल में स्थिर एफ इ सिमुलेशन के माध्यम से क्रैक टिप स्ट्रेस का अनुमान लगाया गया है। ढलान

कोण के साथ क्रैक टिप स्ट्रेस के परिवर्तन पर चर्चा और साहित्य में सूचित परिणामों से तुलना की गयी है। पीएसटी में सीसीएल का अनुमान ऊर्जा आधारित थ्योरी का प्रयोग करके भी लगाया गया है। सीसीएल पर ढलान कोण, स्लैब गुण और कमजोर हिम परत की फ्रैक्चर ऊर्जा के प्रभाव पर चर्चा की गयी है और पूर्व में प्रस्तावित मॉडलों के साथ परिणामों की तुलना भी की गयी है। ढलान कोण के साथ सीसीएल परिवर्तन पर कमजोर हिम परत की डिफ़ोरमेबिलिटी का कोई बड़ा प्रभाव नहीं देखा गया, हालांकि कमजोर कमजोर परत के लिए कम सीसीएल प्राप्त हुई। आदर्श त्रि-विमीय भूभाग और फील्ड पीएसटी में क्रिटिकल क्रैक के आकार पर एक तुलनात्मक अध्ययन भी प्रस्तुत किया गया है।

आइस कॉलम कमजोर हिम परत के साथ पीएसटी के उपरोक्त संख्यात्मक मॉडल का उपयोग हिम परत (स्लैब) के फ्रैक्चर का अध्ययन करने के लिए भी किया गया है। पीएसटी सिमुलेशन में प्राप्त हिम परत (स्लैब) के फ्रैक्चर की प्रकृति तथा एफएसएल पर ऊपरी हिम परत के घनत्व, मोटाई, ढलान कोण आदि के प्रभाव से संबन्धित परिणामों पर चर्चा और फील्ड पीएसटी के परिणामों के साथ तुलना की गयी है। हिम परत (स्लैब) में स्ट्रेस का विश्लेषण और हिम परत (स्लैब) के फ्रैक्चर पर उनकी भूमिका पर भी चर्चा की गई है। हिम के उपरोक्त सातत्य मॉडल का उपयोग फील्ड पीएसटी के सिमुलेशन करने के लिए भी किया गया है तथा सिमुलेशन और फील्ड पीएसटी के परिणामों की तुलना प्रस्तुत की गयी है। हिम के उपरोक्त सातत्यक मॉडल के अनुप्रयोग को आदर्श द्वि और त्रि-विमीय भूभागों में सिमुलेशन के लिए भी विस्तारित किया गया है। कमजोर हिम परत में एफपीएस और ढलान पर क्रैक की स्थिति के सापेक्ष में ऊपर, नीचे और ढलान के आर-पार (केवल 3 विमीय) वाली दिशाओं में फ्रैक्चर प्रसार की दूरी के साथ इसके परिवर्तन को प्रस्तुत किया गया है। द्वि-विमीय भूभाग में क्रैक से ऊपर तथा नीचे की दिशाओं में हिम स्लैब के फ्रैक्चर की प्रकृति और एफएसएल पर ढलान कोण, स्लैब घनत्व और मोटाई के प्रभाव पर चर्चा की गई है। त्रि-विमीय भूभाग में, हिम परत (स्लैब) के फ्रैक्चर का आरंभ प्रारम्भ में उपस्थित क्रैक से ढलान पर ऊपर की ओर से पाया गया जिसका प्रसार बाद में ढलान पर नीचे की ओर और फ्लैंक की तरफ होता हुआ पाया गया।

कुल मिलाकर, उपरोक्त अध्ययन कमजोर हिम परतों में फ्रैक्चर प्रसार की प्रक्रिया और ऊपरी हिम परत (स्लैब) के फ्रैक्चर में काफी अच्छी अंतर्दृष्टि प्रदान करता है। हालाँकि, कम्प्यूटेशनल सीमाओं के कारण, आदर्शित त्रि-विमीय भूभाग में हिम परत (स्लैब) के फ्रैक्चर पर केवल सीमित जानकारी ही प्राप्त होती है।

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List of Symbols

2D	Two dimensional
3D	Three dimensional
a	Crack length
a_c	Critical crack length
A_{cr}	Critical crack area
B	Crack width
β	Slope angle of linear yield surface with hydrostatic pressure axis in p-q plane (Also called as friction angle)
c_e	Elastic wave speed in snow
c_s	Shear elastic wave speed in snow
d	Scalar damage variable
$\dot{d}$	Increment of scalar damage variable
D	Thickness of top snow slab in numerical propagation saw test (PST)
D_b	Thickness of bottom snow slab in numerical PST
DMD	Damage dissipation energy
D_{TS}	Average top slab thickness in natural snow cover
D_{wl}	Thickness of weak snow layer in numerical PST
E	Young's modulus of top snow slab in numerical PST
E_{dyn}	Dynamic Young's modulus of snow
E_{SMP}	Young's modulus of snow estimated using snowmicropen data
E_{snow}	Young's modulus of snow
E_{static}	Static Young's modulus of snow
E_{wl}	Young's modulus of weak snow layer in numerical PST
ε^{pl}	Equivalent plastic strain
ε_0^{pl}	Equivalent plastic strain at the onset of damage
$\dot{\varepsilon}^{pl}$	Increment of equivalent plastic strain
f	Yield function
F_1	Slope parallel component of the slab weight in PST
F_2	Slope perpendicular component of the slab weight in PST
FC	Faceted
g	Gravitational constant

g_1	Slope parallel component of gravitational constant
g_2	Slope perpendicular component of gravitational constant
G	Shear modulus of the top snow slab in numerical PST
G_I	Fracture energy of the weak snow layer in mode-I
G_{II}	Fracture energy of the weak snow layer in mode-II (shear)
G_{fi}	Fracture energy of ice
G_{snow}	Shear modulus of snow
G_{wl}	Shear modulus of weak snow layer in numerical PST
Γ	Energy dissipation in crack extension in the weak snow layer
h	Weak layer collapse height
H	Hardening modulus
K	Kinetic energy of the whole model in numerical PST
k	Ratio of the yield stress in tri-axial tension and tri-axial compression
κ	Timoshenko correction factor for rectangular cross-section
K_{Ic}	Mode-I fracture toughness of homogeneous snow
K_{IIc}	Mode-II fracture toughness of homogeneous snow
l	Characteristic element length
L	Column length in the field or numerical PST
Λ	Characteristic length in weak snow layer
M	Moment due to slope perpendicular component of weight of the slab length over which weak layer is cracked
n	Porosity of snow
ν	Poisson's ratio.
ω_d	Damage initiation parameter
p	Hydrostatic pressure
PD	Plastic dissipation energy
PE	Released potential energy
φ	Dilation angle
ψ	Slope angle
q	Equivalent Mises stress
r_1	Void ratio
r	Third invariant of deviatoric stress
ρ	Density of top snow slab in numerical PST

ρ_{ice}	Ice density
ρ_{snow}	Snow density
ρ_{TS}	Average top slab density in natural snow cover
ρ_{wl}	Density of weak snow layer in numerical PST
S12	Shear stress in continuum weak layer in slope parallel direction in slab-weak layer model
S22	Normal stress in continuum weak layer in slope perpendicular direction in slab-weak layer model
SD	Average density of a snow layer (in stratigraphy data)
ST	Snow type (in stratigraphy data)
σ	Normal stress
$\bar{\sigma}$	Stress in undamaged state
σ_c	Unconfined compression strength of snow
σ_{cmax}	Maximum crack tip compression stress in the weak layer
σ_i	Tensile strength of ice
σ_t	Tensile strength of snow
σ_y	Yield stress
σ_{y0}	Yield stress at the point of damage initiation
t	time
T_{min}	Minimum temperature in natural snow cover
T_{surf}	Snow surface temperature in natural snow cover
τ	Shear stress
τ_{max}	Maximum crack tip shear stress in the weak layer
τ_{max}^b	Maximum crack tip shear stress in the weak layer due to bending of the slab
τ_{max}^{mod}	Modified maximum crack tip shear stress in the weak layer
τ_{max}^t	Maximum crack tip shear stress in the weak layer due to tension in the slab
τ_r	Residual crack face friction stress (shear stress)
τ_s	Shear strength of weak snow layer
θ_3	Angular displacement about axis-3
U_1	Displacement in slope parallel direction
U_2	Displacement in slope perpendicular direction
U_e	Elastic strain energy of the whole model in numerical PST
$\dot{u}^{pl}$	Increment of equivalent plastic displacement
u_f^{pl}	Equivalent plastic displacement at failure

v_s	Volume fraction of ice
v_1	Saw speed
$V(a)$	Crack formation energy
$V_f(a)$	Energy required for weak layer failure over a crack length ‘ a ’
$V_m(a)$	Mechanical energy of Slab-weak layer system with a crack of size ‘ a ’
$V_{mo}(a)$	Mechanical energy of Slab-weak layer system with a short crack
$V_{m1}(a)$	Mechanical energy of Slab-weak layer system with a long crack
w_f	Fracture energy of the weak snow layer
W	Total external work
W_{grav}	Gravitational work
W_{saw}	Work done by the saw
Z	Plastic flow potential

List of abbreviations

AM	Analytical model
CCL	Critical crack length
DEM	Discrete element model
DH	Depth hoar
DP	Drucker-Prager
FEM	Finite element model
FPS	Fracture propagation speed
FSL	Fractured slab length
GS	Grain size (in stratigraphy data)
KE	Kinetic energy
LT	Layer thickness (in stratigraphy data)
MF	Melt-freeze
PERR	Potential energy release rate
PP	Precipitation particles
PST	Propagation saw test
RG	Round grain
SWM	Solitary wave model
TS	Top slab
WC	Wind crust
WL	Weak layer
WLC	Weak layer collapse
WM	Whole model