

**SOME STUDIES ON GRAPH SIGNAL PROCESSING
WITH APPLICATION TO ALZHEIMER'S
DISEASE DETECTION**

HIMANSHU PRAMOD PADOLE



**DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

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by

HIMANSHU PRAMOD PADOLE

DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



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Dedicated to
My Teachers
Family
&
Friends

Certificate

This is to certify that the thesis entitled “**Some Studies on Graph Signal Processing with Application to Alzheimer’s Disease Detection**” being submitted by **Mr. Himanshu Pramod Padole** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of the bonafide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

(Dr. Shiv Dutt Joshi)

(Dr. Tapan Kumar Gandhi)

Professor

Professor

Department of Electrical Engineering Department of Electrical Engineering

Indian Institute of Technology Delhi Indian Institute of Technology Delhi

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Himanshu Pramod Padole

Abstract

In this thesis, we design a novel integrated Alzheimer’s Disease (AD) detection model that exploits both the static and the dynamic brain connectivity based features extracted from resting-state fMRI (rs-fMRI) data to detect AD in the early stages. First, the static connectivity based AD detection model is designed wherein we propose a novel graph frequency based feature extraction method by relating the findings of two neurological experiments. The discriminating graph signal features thus extracted are then used for the classification, which is performed using a properly designed Graph-CNN (GCNN) based graph signal classifier. Performance of this proposed model is then experimentally validated using the rs-fMRI data from ADNI dataset.

Although the aforementioned model classified the normal subjects and the AD patients, with reasonable accuracy, in order to improve its classification performance further, we propose a modification to the existing GCNN architecture, which acts as a graph signal classifier. We design a novel graph wavelet based two-stage multilevel graph coarsening algorithm which is then used to perform the pooling operation in GCNN. The first stage of coarsening uses the graph wavelet based features to coarsen a given graph which is followed by an optimization based second stage, wherein at each level of coarsening, the restriction of the original graph Laplacian is preserved to obtain the reduced graph Laplacian. Efficacy of the proposed coarsening

algorithm is verified in the general context using different graph coarsening quality measures. Its effectiveness as a pooling operator in GCNN is then validated by employing it for the graph coarsening operation in the GCNN architecture. Improvement in the AD classification performance using this modified GCNN architecture attested the superiority of the modified GCNN classifier over the existing approaches for early detection of AD.

Having verified the efficacy of the proposed static connectivity based AD detection model using the rs-fMRI data, we then design a dynamic connectivity based AD detection model which is then integrated with the earlier model to improve the AD detection performance further. We propose a novel approach to characterize the dynamics of the time varying graph connectivity using the state-space representation of the graph signal, wherein the dynamic brain connectivity is modeled as a state of the system while the input graph signal serves as an observation. The dynamics of the time varying graph connectivity is characterized by the resulting state transition matrix which is then used as a dynamic connectivity based feature for AD classification purpose. After verifying the utility of this dynamic connectivity based AD detection model, we integrate it with our static connectivity based AD detection model discussed earlier using multimodal deep learning architecture. State-of-the-art classification performance of this modified AD detection model corroborated its efficacy in AD detection application.

सार

इस शोध प्रबंध में हमने एक अभिनव एकीकृत अल्जायमर डिजीज (AD) अनुसन्धान का प्रतिमान अभिकल्पित किया है, जो rs-fMRI डेटा से प्राप्त किए स्थिर एवं गतिशील मस्तिष्क संयोजकता पर आधारित लक्षणों से प्रारंभिक अवस्था में ही अल्जायमर का निदान कर लेता है | प्रथमतः एक स्थिर संयोजकता पर आधारित AD अनुसन्धान का प्रतिमान अभिकल्पित किया गया है, जिसमें हम दो स्नायविक प्रयोगों के निष्कर्षों को जोड़कर एक अभिनव ग्राफ आवृत्ति आधारित लक्षण प्राप्त करने की विधि प्रस्तावित करते हैं | इस प्रकार निकाले गए विभेदक ग्राफ सिग्नल लक्षणों का उपयोग एक उचित रूप से अभिकल्पित GCNN आधारित ग्राफ सिग्नल वर्गीकारक के निर्माण के लिए किया गया है | इस प्रस्तावित प्रतिमान का प्रदर्शन फिर ADNI डेटासेट से प्राप्त rs-fMRI डेटा का उपयोग करके प्रयोगात्मक रूप से सत्यापित किया गया है |

यद्यपि उपरोक्त प्रतिमान ने स्वस्थ व्यक्ति और AD रोगियों को उचित सटीकता के साथ वर्गीकृत किया, अपने वर्गीकरण प्रदर्शन को श्रेष्ठतर बनाने के लिए हम वर्तमान GCNN आर्किटेक्चर में संशोधन का प्रस्ताव करते हैं, जो की एक ग्राफ सिग्नल वर्गीकारक के रूप में कार्य करता है | हम एक अभिनव ग्राफ wavelet-आधारित द्विचरण बहुस्तरीय ग्राफ coarsening प्रणाली अभिकल्पित करते हैं जिसका उपयोग फिर GCNN में पूलिंग कार्य करने के लिए किया गया है | Coarsening के प्रथम चरण में ग्राफ वेवलेट आधारित लक्षणों का उपयोग ग्राफ को कोर्सन करने के लिए किया गया है | इसके बाद एक इष्टतमीकरण आधारित दुसरा चरण होता है, जिसमें प्रत्येक स्तर की कोर्सनिंग में मूल ग्राफ लाप्लासियन के प्रतिबंध को संरक्षित रखते हुए coarsened ग्राफ लाप्लासियन प्राप्त करते हैं | प्रस्तावित कोर्सनिंग प्रणाली की सटीकता को सामान्य संदर्भ में विभिन्न ग्राफ कोर्सनिंग परिमाणों का उपयोग करके सत्यापित किया गया है | GCNN में पूलिंग प्रचालक के रूप में इसकी परिणामकारकता को प्रमाणित करने के लिए इसका उपयोग फिर

GCNN में ग्राफ कोर्सनिंग कार्य के लिए नियोजित किया गया है | संशोधित GCNN आर्किटेक्चर की AD निदान कार्य में श्रेष्ठतर वर्गीकरण सटीकता इसकी AD निदान कार्य में श्रेष्ठता को प्रमाणित करता है |

प्रस्तावित स्थिर संयोजकता आधारित AD अनुसन्धान प्रतिमान की सटीकता को rs-fMRI डेटा से सत्यापित करने के पश्चात् हम एक गतिशील संयोजकता आधारित AD अनुसन्धान प्रतिमान अभिकल्पित करते हैं | AD निदान कार्य की सटीकता बढ़ाने के लिए हम इसे फिर हमारे पूर्व प्रस्तावित AD अनुसन्धान प्रतिमान के साथ एकीकृत करते हैं | हम एक अभिनव state-space आधारित पद्धति का प्रस्ताव करते हैं, ज्यो ग्राफ संयोजकता की गतिशीलता को चिन्हीत करता है, जिसमें गतिशील मस्तिष्क संयोजकता सिस्टीम की state के रूप में कार्य करती जाता है, जबकि निविष्ट ग्राफ सिग्नल एक observation के रूप में कार्य करता है | ग्राफ संयोजकता की गतिशीलता को स्टेट ट्रांज़िशन मॅट्रिक्स चिह्नित करता है, जिसका फिर AD वर्गीकरण के लिए एक गतिशील संयोजकता आधारित लक्षण के रूप में उपयोग किया जाता है | इस गतिशील संयोजकता आधारित प्रतिमान की उपयोगिता को सत्यापित करने के पश्चात् हम मल्टीमॉडेल डीप लर्निंग आर्किटेक्चर का उपयोग करते हुए इसे फिर पूर्व चर्चित स्थिर संयोजकता आधारित AD अनुसन्धान प्रतिमान साथ एकीकृत करते हैं | इस एकीकृत AD अनुसन्धान प्रतिमान का श्रेष्ठतर वर्गीकरण प्रदर्शन इसकी AD निदान कार्य में उपयोगिता की पुष्टि करता है |

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List of Notations and Abbreviations

\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
$(\cdot)^T$	Transpose
\mathcal{V}^c	Complement of \mathcal{V}
\odot	Hadamard product
$\ \cdot\ _2$	l_2 norm
$\ \cdot\ _F$	Frobenius norm
AD	Alzheimer's Disease
MCI	Mild Cognitive Impairment
GSP	Graph Signal Processing
MRI	Magnetic Resonance Imaging
fMRI	functional Magnetic Resonance Imaging
rs-fMRI	resting-state functional Magnetic Resonance Imaging
BOLD	Blood Oxygen Level Dependent

PET	Positron Emission Tomography
EEG	Electroencephalography
CNN	Convolutional Neural Network
GCNN	Graph Convolutional Neural Network
GFT	Graph Fourier Transform
GWT	Graph Wavelet Transform
SSM	State-Space Model
ADNI	Alzheimer's Disease Neuroimaging Initiative
HPF	High Pass Filter
NC	Normal Control
SVM	Support Vector Machine