A THESIS ON
SINGLE STEP METHOD FOR INITIAL VALUE PROBLEMS IN DIFFERENTIAL EQUATIONS

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C E R T I F I C A T E

This is to certify that the thesis entitled 'Single-Step Methods For Initial-Value Problems In Differential Equations' which is being submitted by Miss Usha Ahluwalia for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. She has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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S Y N O P S I S

This thesis attempts to provide some single-step methods for the numerical integration of ordinary and partial differential equations. The complete discussion has been carried out in four chapters. The first chapter gives an exposition of the current stage of the one-step methods available in the research literature for the numerical solution of differential equations. The second and third chapters deal with the Goursat problem for the hyperbolic partial differential equations. The fourth chapter deals with the non-explicit Runge-Kutta methods for the numerical solution of first order differential equations.

Chapter - I

Many numerical methods are available for the approximate solution of ordinary and partial differential equations. This chapter gives an exposition of some of these methods and points out the main draw-backs in them. It further suggests some possible improvements to obtain methods better in some sense for the numerical integration of differential equations.

Chapter - II

This chapter has been divided into two parts. Part-I deals with the higher order Hermite cubature formulae and Part-II deals with the application of these formulae to the Goursat problem for the hyperbolic partial differential equations.

Part-I: In this part, with the help of a transformation of the dependent variable, we have obtained Hermite cubature formulae of arbitrary order. The section is closed with a numerical example where the results obtained by this method have been compared with the exact results. The approximate solution was found to be in good agreement with the exact solution.

Part-II: The method described in this section can be used to find the numerical solution to the Goursat problem in hyperbolic partial differential equations. The method uses Hermite cubature formulae derived in Part-I to find the numerical solution with arbitrary order of accuracy.to the Goursat problem..

Non-linear examples have been computed to show the comparison with the exact solution.

Chapter - III

In this chapter, we have developed a numerical procedure with constrained error bounds, based on that of Moore, for finding approximate solution of the Goursat problem in hyperbolic partial differential equations. This has been done by imposing certain conditions and constraints, which reduce the error term considerably.

Sample examples have been computed to show the accuracy of the computed results.

Chapter - IV

In this chapter, we have modified the transformation given by Fehlberg and obtained non-explicit Runge-Kutta formulae upto order (m+7) for the case of first order differential equation. The stability region has been obtained for various values of m.

A set of examples have been computed to illustrate the comparison of the results obtained from the non-explicit formulae with the exact solution.

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