

STATISTICAL UNCERTAINTY QUANTIFICATION IN POWER SYSTEM SUBJECTED TO STOCHASTIC INJECTIONS

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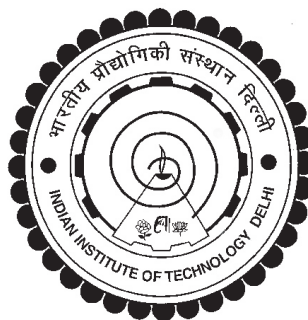
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Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



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I dedicate this thesis to my loving family, friends and all my teachers

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements.

Suravi Thakur

June 2024

Certificate

This is to certify that the dissertation entitled '**Statistical Uncertainty Quantification in Power Systems subjected to Stochastic Injections**', being submitted by **Mrs. Suravi Thakur** for the award of the degree of **Doctor of Philosophy** is a record of bonafide research work carried out by her in the Department of Electrical Engineering at Indian Institute of Technology Delhi, New Delhi.

Mrs. Suravi Thakur has worked under my supervision and has fulfilled the requirements for the submission of this dissertation, which to my knowledge has reached the requisite standard. The results obtained here have not been submitted to any other University or Institute for the award of any degree.

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Abstract

Owing to the increased penetration of intermittent energy sources and highly flexible power demands, the power system dynamics is experiencing an increasing order of uncertainty. Proper modeling and quantification of these uncertainties are essential for obtaining a more realistic behaviour of power systems. Ignoring the impact of uncertainties may lead to incorrect decision-making in power system operation, planning and security assessment. The uncertainties which the power system can experience can be broadly classified into the following categories: *Parameter Uncertainty* (e.g. lack of knowledge of all the parameters for loads, renewable generations, transmission models etc.), *Observational and Measurement Uncertainty* (e.g. PMU, SCADA), *Model Inaccuracy Uncertainty* (e.g. simplified and equivalent models of generators), *Parameter Variation and External Uncertainty* (atmospheric, man-made or industrial).

A deterministic approach is generally used for power system analysis. Such an approach analyses the power system for a specific scenario, ignoring the impact of uncertainties on the states and parameters. The operational decisions for system security and stability are taken based on the worst-case scenarios only [1]. On the other hand, a probabilistic approach considers the probability distribution of all the uncertainties and observes its impact on the overall performance of the power system. A lot of work has been done to model these uncertainties appropriately. Broadly, two modelling approaches have been used i) Static random variable based and ii) Continuous random process based. While the modelling of these uncertainties has gained increased attention in recent years, the proper analysis of the resulting data for characterising the impact of these uncertainties on the power system

variables is equally important.

Probabilistic approaches are, however, associated with higher computational burden and mathematical complexities. This can be overcome in two ways. Firstly, by performing the Uncertainty Quantification (UQ) only for the most critical uncertainties in the power systems, thereby limiting the modeling burden and secondly by using efficient and appropriate sampling techniques before the analysis.

An appropriate sampling based Random Variable approach has been proposed in this thesis for carrying out Statistical Uncertainty Quantification of the power system with randomness. Such an approach will provide more accurate results for overall power system performance analysis, including demand side management, power system planning, control and operations for future power systems, which will have increased penetration of renewable energy sources and highly flexible load demands, leading to both spatial and temporal uncertainties. The proposed methodologies have been tested on benchmark systems like IEEE-9 bus system, IEEE-68 bus system and Kundur's two area system. The power demand and frequency data obtained from Northern Region Load Dispatch Centre (NRLDC), India has also been used to validate the test results.

सारांश

अंतरालिक ऊर्जा स्रोतों की बढ़ती पैठ और अत्यधिक लचीली बिजली की मांगों के कारण, विद्युत प्रणाली की गतिशीलता में अनिश्चितता का बढ़ता हुआ स्तर देखने को मिल रहा है। इन अनिश्चितताओं का सही मॉडलिंग और मात्राकरण विद्युत प्रणालियों के यथार्थवादी व्यवहार को प्राप्त करने के लिए आवश्यक है। अनिश्चितताओं के प्रभाव को अनदेखा करना विद्युत प्रणाली के संचालन, योजना और सुरक्षा आकलन में गलत निर्णय लेने की स्थिति पैदा कर सकता है। विद्युत प्रणाली जिन अनिश्चितताओं का सामना कर सकती है, उन्हें व्यापक रूप से निम्नलिखित श्रेणियों में वर्गीकृत किया जा सकता है: **पैरामीटर अनिश्चितता** (जैसे लोड्स, नवीकरणीय उत्पत्तियों, ट्रांसमिशन मॉडलों के सभी पैरामीटर का ज्ञान न होना), **प्रेक्षण और मापन अनिश्चितता** (जैसे PMU, SCADA), **मॉडल की अशुद्धि अनिश्चितता** (जैसे जनरेटर के सरलीकृत और समतुल्य मॉडल), **पैरामीटर भिन्नता और बाहरी अनिश्चितता** (वायुमंडलीय, मानव निर्मित या औद्योगिक)।

सामान्यतः विद्युत प्रणाली विश्लेषण के लिए एक निर्धारक दृष्टिकोण का उपयोग किया जाता है। ऐसा दृष्टिकोण विद्युत प्रणाली का विश्लेषण एक विशिष्ट परिदृश्य के लिए करता है, जिसमें राज्यों और पैरामीटर पर अनिश्चितताओं के प्रभाव को नजरअंदाज किया जाता है। प्रणाली की सुरक्षा और स्थिरता के लिए संचालन निर्णय केवल सबसे खराब स्थिति पर आधारित होते हैं। दूसरी ओर, एक प्रायिकता दृष्टिकोण सभी अनिश्चितताओं के प्रायिकता वितरण पर विचार करता है और इसका प्रभाव विद्युत प्रणाली के कुल प्रदर्शन पर देखता है। इन अनिश्चितताओं को उचित रूप से मॉडल करने के लिए बहुत काम किया गया है। व्यापक रूप से, दो मॉडलिंग दृष्टिकोण उपयोग किए गए हैं i) स्थैतिक रैंडम वेरिएबल आधारित और ii) सतत रैंडम प्रक्रिया आधारित। हालांकि इन अनिश्चितताओं के मॉडलिंग ने हाल के वर्षों में अधिक ध्यान प्राप्त किया है, विद्युत प्रणाली चर पर इन अनिश्चितताओं के प्रभाव को चित्रित करने के लिए परिणामी डेटा का उचित विश्लेषण समान रूप से महत्वपूर्ण है।

हालांकि प्रायिकता दृष्टिकोण उच्च गणनात्मक भार और गणितीय जटिलताओं से जुड़े होते हैं। इसे दो तरीकों से दूर किया जा सकता है। पहला, विद्युत प्रणालियों में सबसे महत्वपूर्ण अनिश्चितताओं के लिए ही अनिश्चितता मात्राकरण (UQ) करने से, जिससे मॉडलिंग बोझ को सीमित किया जा सके और दूसरा, विश्लेषण से पहले प्रभावी और उचित सैंपलिंग तकनीकों का उपयोग करके।

इस शोध में सांख्यिकीय अनिश्चितता मात्राकरण के लिए उपयुक्त सैंपलिंग आधारित रैंडम वेरिएबल दृष्टिकोण प्रस्तावित किया गया है, जिसमें अनिश्चितता के साथ विद्युत प्रणाली का विश्लेषण किया गया है। ऐसा दृष्टिकोण भविष्य की विद्युत प्रणालियों के प्रदर्शन विश्लेषण के लिए अधिक सटीक परिणाम प्रदान करेगा, जिसमें मांग पक्ष प्रबंधन, विद्युत प्रणाली योजना, नियंत्रण और संचालन शामिल हैं, जो नवीकरणीय ऊर्जा स्रोतों की बढ़ती पैठ और अत्यधिक लचीली लोड मांगों के कारण स्थानिक और सामयिक अनिश्चितताओं का सामना

करेंगे। प्रस्तावित पद्धतियों का परीक्षण आईईईई-9 बस प्रणाली, आईईईई-68 बस प्रणाली और कुंदुर के दो क्षेत्र प्रणाली जैसे मानक प्रणालियों पर किया गया है। उत्तरी क्षेत्र लोड डिस्पैच सेंटर (NRLDC), भारत से प्राप्त बिजली की मांग और आवृत्ति डेटा का उपयोग भी परीक्षण परिणामों को सत्यापित करने के लिए किया गया है।

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Nomenclature

Abbreviations

AA	Affine Arithmetic
AIC	Akaike's Information Criterion
BIC	Bayesian Information Criteria
CSI	Channel State Information
EM	Expectation Maximization
GMM	Gaussian Mixture Model
ICT	Information and Communications Technology
IID	Identically Distributed and Independent
K-S	Kolmogorov-Smirnov
MC	Monte Carlo
NRLDC	Northern Regional Load Dispatch Centre
OUP	Ornstein Uhlenbeck Process
PC	Polynomial Chaos
PCM	Probabilistic Collocation Method

PIVP	Probabilistic Initial Value Problem
PV	Photovoltaic
QQ	Quantile-Quantile
RE	Renewable Energy
RLF	Repeated Load Flow
SDAE	Stochastic Differential Algebraic Equations
SDE	Stochastic Differential Equations
SSS	Small Signal Stability
TDS	Time Domain Simulation
UQ	Uncertainty Quantification
VLSI	Very Large Scale Integration
WSS	Wide Sense Stationary
WTG	Wind Turbine Generator