

OPTIMAL MAINTENANCE POLICIES UNDER INCOMPLETE STATE INFORMATION

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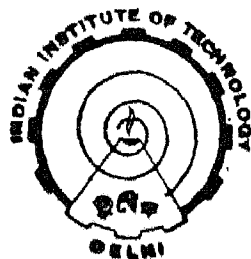
by

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SUBMITTED

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CERTIFICATE

This is to certify that the thesis entitled OPTIMAL MAINTENANCE POLICIES UNDER INCOMPLETE STATE INFORMATION which is being submitted by Mr.Sraban Mukherjee to the Indian Institute of Technology, Delhi , for the award of the Degree of Doctor of Philosophy , is a record of bonafide research work carried by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis, which has attained the standard required for a Ph.D. degree of this institute. This work has not been submitted elsewhere for the award of any degree or diploma.



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(SRABAN MUKHERJEE)

ABSTRACT

Over the last decade there has been extensive research into the control problem of Markov processes for which only partial or incomplete state information is available. This type of problem is commonly known as the Partially Observed Markov Decision Process (POMDP) problem. Applications of POMDP include many varied areas such as quality control, machine maintenance, search strategies, teaching strategies, noise-corrupted data communication, investment decision and so on. In machine replacement problems there are many situations where the real state of the deteriorating system is not known with certainty, but a certain type of inspection is carried out to estimate the extent of deterioration. In general the problem structure of the POMDP is very large ranging from a completely observed process to a completely unobserved process depending on the quality of information received during inspection.

A POMDP is a generalisation of a Markov Decision Process (MDP) with an enlarged state space consisting of the space of probability distributions over the underlying core state. Hence the computation procedure for finding optimal control policies is more complex than its MDP counterpart. In POMDP models there is an element of uncertainty regarding the actual state of the system which has significant impact on the structure of the optimal policy.

A modified algorithm is developed for calculating optimal policies for finite horizon partially observed Markov processes. This modification covers two main aspects: (a) the correction of a serious flaw in the only available algorithm for the finite

horizon problem given by Smallwood and Sondik [35], and (b) the improvement of the efficiency of the algorithm by identifying and eliminating redundancies which generally exist in large numbers.

Sufficient conditions are presented for the POMDP optimal value function to be monotone on the space of state probability vectors. The thesis also presents sufficient conditions for the optimal policy of a simple replacement problem to be monotone.

The application of the POMDP model is extended to a production process problem in which it has been assumed that the system can be repaired at a cost lower than that of replacement. It has also been assumed that the decision-maker has partial information regarding the state of the system and to know the exact state costly inspection is required. The problem is to minimize the expected total discounted cost over an infinite horizon. It has been shown that there exists a six-region monotonic optimal policy.

A POMDP model is developed for a machine maintenance problem where it has been assumed that the spares are not readily available. It has been shown that the optimal ordering, inspection and replacement policy has at the most four-region under certain conditions on the system parameters.

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