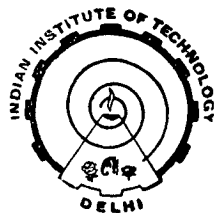


**COUPLED BENDING VIBRATION CHARACTERISTICS  
OF  
PRE-TWISTED ROTATING BLADES**

By  
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A THESIS  
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CERTIFICATE

This is to certify that the thesis entitled "COUPLED BENDING VIBRATION CHARACTERISTICS OF PRETWISTED ROTATING BLADES" by Varuna Reddy Potula has been prepared under our supervisions in conformity with the rules and regulations of the Indian Institute of Technology, New Delhi. We further certify that the thesis has attained a standard required for a Ph.D. degree of the Institute. The results contained in this thesis has not been submitted, in part or full, to any other university for any degree or diploma.

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(P.V. Reddy)

## A B S T R A C T

The work presented in this thesis pertains to coupled bending-bending vibration of turbomachine blades. A turbomachine blade is considered as a pretwisted cantilever blade of asymmetric cross-section mounted on a rotating disc at zero stagger angle. The pretwist of the blade causes coupling in both the bending directions. The asymmetry of the cross-section couples the torsional motion of the blade with bending. But this effect is known to be very small, and so it is ignored. Second order effects such as shear deflection and rotary inertia, Coriolis forces are also neglected.

The coupled bending-bending frequencies and corresponding modes are obtained by the method of polynomial frequency equation. This method has the advantage over the conventional transfer matrix in the sense that it eliminates the iterative procedure method of matrix multiplication for frequency determination. Deak and Baird (1963) equations for pretwisted blade are used to evaluate the polynomial frequency equation.

The expressions for polynomial coefficients are first developed for stationary pretwisted blade and then extended to the rotating blade. A general computer program is developed to evaluate the coefficients of polynomial frequency equation for  $n$ , number of stations. The programme also evaluates the roots i.e. the natural frequencies and corresponding mode shapes of the given blade.

The results obtained in this work are in good agreement with those available in the literature and also with experimental values obtained for rotating pretwisted blades. Thus the validity of the generalised expressions for the coefficients is established.

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