

**A METHODOLOGY FOR SIMULTANEOUS BUS TRANSIT  
ROUTE NETWORK DESIGN AND FREQUENCY SETTING  
PROBLEM IN SMALL AND MEDIUM SIZED CITIES**

SEYED MOHAMMAD HASSAN MAHDAVI MOGHADDAM



DEPARTMENT OF CIVIL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI

**OCTOBER 2016**

©Indian Institute of Technology Delhi (IITD), New Delhi, 2016

**A METHODOLOGY FOR SIMULTANEOUS BUS TRANSIT  
ROUTE NETWORK DESIGN AND FREQUENCY SETTING  
PROBLEM IN SMALL AND MEDIUM SIZED CITIES**

by:

**SEYED MOHAMMAD HASSAN MAHDAVI MOGHADDAM**

Department Of Civil Engineering

Submitted

In fulfillment of the requirements of the degree of **Doctor of Philosophy**

to the



**Indian Institute of Technology Delhi**

**OCTOBER 2016**

*To my Father and Mother*

# Certificate

This is to certify that the thesis entitled, ‘**A methodology for simultaneous bus transit route network design and frequency setting problem in small and medium sized cities**’ being submitted by **Mr. Seyed Mohammad Hassan Mahdavi Moghaddam** to the **Indian Institute of Technology Delhi**, is a record of the bona-fide research work carried out by him under our supervision. The thesis, in our opinion, is worthy of consideration for the award of the degree of **Doctor of Philosophy** in accordance with the regulations of the Institute. The results embodied in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Dr. Kalaga Ramachandra Rao  
Associate Professor  
Department of Civil Engineering  
Indian Institute of Technology Delhi  
New Delhi, India 110 016

Dr. Geetam Tiwari  
Professor  
Department of Civil Engineering  
Indian Institute of Technology Delhi  
New Delhi, India 110 016

Date:  
New Delhi

# Acknowledgments

As any researcher comes to realize, it is impossible to acknowledge all of the many people who have contributed to their understanding of any given subject, nor to recognize everyone who is deserving of mention. Still, there are many people whose unique contributions need to be mentioned. Foremost among them I would like to express my deepest gratitude to Dr. Kalaga Ramachandra Rao and Dr. Geetam Tiwari under whose guidance this research was carried out. I am thankful to them for the given motivation and support. I especially thank them for the freedom and encouragement they have given me from the very beginning of my work that shaped my research.

My sincere thanks to Dr. Pravesh Biyani from Indraprastha Institute of Information Technology Delhi with whom I had many fruitful discussions on optimization problems. I would like to thank Dr. Dangeti Mukund Rao from GITAM University, Vishakhapatnam, India, who helped me in understanding detail transit operation problems. I also would like to thank my colleague in the Institute and program manager at Shakti Sustainable Energy Foundation, Mr. Ravi Gadepalli who provided me useful data and information regarding Vishakhapatnam city.

My special thanks go to my beloved Father, Mother and Sisters who always supported and encouraged me with their best wishes. They were always there cheering me up and support me in good and bad days. I would like to thank my wife, Fatemeh who came to my life during the days of Ph.D., for not only putting up with my endless hours of sitting at my computer, but also encouraging me with love and patience. All through the period of research my parent has given me the much needed emotional support and inspiration, for which I am greatly indebted. I dedicate this work to them as a token of deep love. Above all, I glorify my God, the most gracious, the most merciful, for his blessings.

*October 2016*

*Seyed Mohammad Hassan Mahdavi Moghaddam*

# Abstract

The procedure of bus route network design that is a primary phase of public transit planning operation has a significant influence on the overall performance of operation. The design for bus passenger's movement in the real network is a complex non convex problem typically being NP-hard involving optimization of several parameters under some complex constraints. In reality, efficient procedure of utilizing the resources does not only depend on variations related to user's and operator's perspectives but also assessment of available infrastructure, analysis of input data, external effects of operation on the users and mathematical modelling. Hence, its efficiency and optimum performance depend on the way problem is explained. A proposed simultaneous bus route network design and frequency setting procedure is carried out in five phases. (1) The input analysis phase has the following sub-phases; (a) multiple objective function to minimize user, operator and pollution costs subject to constraints, (b) delineation of public transit traffic analysis zones and passenger travel demand matrix for bus stop level and (c) generation of operating network that has following sub-phases; (a) non-clustered operating network and (b) clustered operating network, (2) The generation phase has the following sub-phases; (a) path generation and filtration where different filtration rules are carried out to identify feasible regions within search space. Next, the performance, optimization and evaluation phases work all together in assessment procedure. (3) The route performance phase assesses generated candidate routes through the following sub-phases; (a) feasibility and connectivity check, (b) iterative passenger demand assignment procedure considering multipath effects. (4) The optimization phase generates initial solution from generated solution space and identifies an optimum set of routes with minimum total operations cost using genetic algorithm. A transfer minimization operator is implemented within genetic algorithm procedure to further increase the quality of solution and minimize the number of transfers and

randomness of GA. (5) The evaluation phase works in-between the performance and the optimization phases. It evaluates optimum route sets according to related service frequencies (identified in performance phase). The proposed model is applied to small and large size test networks to investigate its efficiency and performance. Mandl's Swiss network is used as a benchmark network. Results confirm the efficiency of the proposed model through set of quality indicators. A bus transit network of Vishakhapatnam city as a small and medium sized Indian city is selected to further investigate the performance of proposed model for large size network. Results indicate improvement over the quality of operations compared to existing one on both clustered and non-clustered operating networks.

**Keywords:** simultaneous bus route network design, bus frequency setting, delineation of public transit traffic analysis zones, route generation, genetic algorithm, small and medium sized city, transfer minimization operator

# Table of Contents

<b>CERTIFICATE</b>	<b>I</b>
<b>ACKNOWLEDGMENTS</b>	<b>III</b>
<b>ABSTRACT</b>	<b>V</b>
<b>TABLE OF CONTENTS</b>	<b>VII</b>
<b>LIST OF FIGURES</b>	<b>XI</b>
<b>LIST OF TABLES</b>	<b>XIII</b>
<b>NOTATION</b>	<b>XVII</b>
<b>1. INTRODUCTION</b>	<b>1</b>
1.1. Indian small and medium sized city’s public transportation	2
1.2. Motivation	3
1.3. Objectives and Scope	6
1.4. Methodology	8
1.5. Organization of thesis	15
<b>2. STATE OF THE ART</b>	<b>17</b>
2.1. Bus route network design and frequency setting components	18
2.1.1. Multi-objective function in bus route network design problem	19
2.1.1.1. Weighted sum converting to single objective approach	20
2.1.1.2. Bi-level Mixed integer programming	20
2.1.1.3. Pareto optimality as multi-objective evolutionary approach	21
2.1.2. Variation of constraints in bus route network design problems	22
2.1.3. Parameters	24
2.1.3.1. Effect of demand estimation models in bus route network design problem	25
2.1.3.2. Variable demand versus fixed demand in bus route network design	25
2.1.3.3. Traffic analysis zones and passenger travel demand	26
2.1.4. Demand assignment methods	27
2.1.5. Route Construction procedures	28
2.1.5.1. Shortest path algorithms	28
2.1.6. Optimization algorithms	29
2.1.7. Discussion	31

2.2.	Heuristic approaches in bus route network design	33
2.3.	Meta-heuristic approaches in bus route network design	34
2.4.	Conclusions	40
<b>3.</b>	<b>MODEL FORMULATION</b>	<b>43</b>
3.1.	Context of planning Model	43
3.1.1.	Multi-objective function formulation	45
3.1.2.	User cost	46
3.1.3.	Operator cost	47
3.1.3.1.	Running Cost of operation	49
3.1.3.2.	Fixed and maintenance cost of operation	49
3.1.4.	External pollution cost	49
3.1.5.	Formulation of weighted sum multi-objective function	52
3.1.6.	Formulation of constraints	54
3.2.	Bus passenger travel demand	59
3.2.1.	Evaluation of modes of travel – Case study of Vishakhapatnam city	60
3.2.1.1.	Willingness to shift from 3-wheeler and shared 3-wheeler to buses	62
3.2.1.2.	Willingness to shift from Buses to 2-wheelers	62
3.2.1.3.	Willingness to shift from 2-wheeler to buses	63
3.2.1.5.	Total potential bus transit demand	64
3.3.	Delineation of traffic analysis zones	66
3.4.	Generation of the operating network	81
3.4.1.	Sparse distribution of passenger travel demand	84
3.4.2.	Network clustering threshold	85
3.4.3.	Identification of cluster centroids	86
3.5.	Conclusions	89
<b>4.</b>	<b>CLUSTERED AND NON-CLUSTERED BUS ROUTE NETWORK DESIGN AND FREQUENCY SETTING USING GENETIC ALGORITHM</b>	<b>91</b>
4.1.	Mathematical Complexity - Global search space	92
4.1.1.	Exhaustive passenger demand assignment procedure (EPDA)	93
4.2.	Generation phase - Path generation and filtration	97
4.2.1.	Route generation algorithm (RGA)	98
4.2.2.	Route filtration algorithm (RFA)	99
4.3.	Optimization phase - Genetic algorithm for BRND-FS problem	106
4.3.1.	Representation of variables in genetic algorithm	108
4.3.2.	Transfer minimization operator in genetic algorithm	113
4.3.3.	Iterative demand assignment and frequency setting algorithm	117
4.3.3.1.	Initial minimum service frequency setting	117

4.3.3.2.	Multi-path assignment procedure and frequency setting	118
4.3.4.	Model assumptions	121
4.4.	Conclusions	122
<b>5.</b>	<b>STUDY ON SAMPLE NETWORKS</b>	<b>123</b>
5.1.	Benchmarking	124
5.1.1.	Quality indicators	125
5.1.2.	Set of feasible routes and service frequencies for different number of operating routes	127
5.2.	Case study of large network - Vishakhapatnam city, India	134
5.2.1.	Characteristics of bus transit operation in Vishakhapatnam city	135
5.2.2.	Existing bus route network in the city	136
5.2.3.	Generation, performance and Optimization phases	137
5.2.3.1.	Operation cost	138
5.2.3.2.	Pollution cost	139
5.2.3.3.	Operating network	141
5.2.3.4.	Candidate route generation phase	142
5.2.3.5.	Performance, optimization and evaluation phases	143
5.2.3.6.	Comparison of results through performance measurement	144
5.2.4.	Financial viability	147
5.3.	Conclusions	149
	<b>CONCLUSIONS</b>	<b>151</b>
6.1.	Summary	152
6.2.	Suggestions for future work	155
	<b>REFERENCES</b>	<b>157</b>
	<b>APPENDIX 1. DEVELOPMENT OF BUS ROUTE NETWORK DESIGN AND FREQUENCY SETTING SOFTWARE</b>	<b>163</b>
	<b>BIO-DATA</b>	<b>167</b>

# List of Figures

FIGURE 1.1. COMPONENTS OF BUS ROUTE NETWORK DESIGN AND FREQUENCY SETTING .....	5
FIGURE 1.2. PROPOSED METHODOLOGY FOR BRND-FS .....	14
FIGURE 2.1. DIFFERENT OBJECTIVE FUNCTIONS IN BRND .....	19
FIGURE 3.1 UNDIRECTED GRAPH (SAMPLE NETWORK) .....	44
FIGURE 3.2 CHARACTERISTICS OPERATION OF PASSENGER MOVEMENT FROM ORIGIN TO DESTINATION .....	45
FIGURE 3.3 DETERMINATION OF TOTAL TRAVEL DEMAND .....	61
FIGURE 3.4. DETAILED NETWORK ELEMENTS (A. ROADS, B. OPERABLE BUS TRANSIT ROADS) .....	71
FIGURE 3.5. DATA ALLOCATION FOR CHARACTERIZATION OF TAZ.....	72
FIGURE 3.6. PARTITIONING OF EXISTING TAZ INTO CELL GRIDS (PTTAZ), CENTROID AND CENTROID CONNECTORS .....	73
FIGURE 3.7. RESIDENTIAL AND COMMERCIAL BLOCKS FOR ATTRACTION AND PRODUCTION MAPPING IN EACH PTTAZ GRID CELL.....	74
FIGURE 3.8. CELL GRID SUBZONE (PTTAZ) POPULATION DENSITY RATIO ESTIMATION .....	76
FIGURE 3.9. ACCESSIBILITY MEASURE FOR DEMAND ALLOCATION FROM (PTTAZ) CELL GRIDS TO BUS STOPS.....	77
FIGURE 3.10. COMPARING AVERAGE TRAVEL TIME IN TAZ AND PTTAZ SCHEMES.....	79
FIGURE 3.11 PROCEDURE FOR CLUSTERING THE OPERATING NETWORK WITH SPARSE MATRIX .....	83
FIGURE 3.12 NODE CHARACTERISTIC IDENTIFICATION .....	84
FIGURE 3.13. NETWORK ADJACENCY (VISHAKHAPATNAM CITY).....	85
FIGURE 3.14 ACCESSIBILITY BASED IN EUCLIDIAN DISTANCE AND NETWORK BASED DISTANCE IMPLEMENTED IN GIS (VISHAKHAPATNAM CITY) .....	86
FIGURE 4.1. UNDIRECTED GRAPH (SAMPLE NETWORK 2) .....	94
FIGURE 4.2 CHOICES AVAILABLE TO PASSENGER TRIP .....	95
FIGURE 4.3. MATLAB PARALLEL COMPUTING PROCEDURE FOR EXHAUSTIVE SEARCH METHOD .....	97
FIGURE 4.4 REMOVAL OF OVERLAPPING ROUTES IN ROUTE FILTRATION ALGORITHM.....	101
FIGURE 4.5. MANDL'S SWISS NETWORK .....	102
FIGURE 4.6. PROPOSED PROCEDURE FOR SIMULTANEOUS OPTIMIZATION OF ROUTES AND FREQUENCIES USING GA .....	107
FIGURE 4.7. GLOBAL SEARCH SPACE AND SELECTION OF RANDOM POPULATION INDIVIDUALS IN GA.....	108
FIGURE 4.8. CODING OF ROUTES, SERVICE FREQUENCIES AND CORRESPONDING NODES .....	109

FIGURE 4.9. TWO POINT CROSSOVER APPLICATION FOR EIGHT ROUTES.....	112
FIGURE 4.10. SINGLE POINT MUTATION APPLICATION FOR EIGHT ROUTES .....	112
FIGURE 4.11. CANDIDATE INDIVIDUAL SELECTION PROCEDURE IN TMO .....	115
FIGURE 4.12. CANDIDATE INDIVIDUAL EVALUATION PROCEDURE IN TMO .....	116
FIGURE 5.1. MANDL’S SWISS NETWORK .....	124
FIGURE 5.2. CHANGE OF INDIRECT DEMAND RATIO, OVER SET OF DIFFERENT ROUTE SIZES .....	131
FIGURE 5.3. CHANGE OF AVERAGE TRAVEL TIME OVER SET OF DIFFERENT ROUTE SIZES .....	132
FIGURE 5.4. CHANGE OF FLEET SIZE OVER SET OF DIFFERENT ROUTE SIZES .....	132
FIGURE 5.5. EXISTING OPERATING BUS ROUTES IN VISHAKHAPATNAM CITY .....	137
FIGURE 5.6. VISHAKHAPATNAM GVMC BOUNDARY WITH EXISTING BUS STOPS, OPERABLE ROAD NETWORK.....	138
FIGURE 5.7. LARGE SIZE OPERATING NETWORK OF VISHAKHAPATNAM CITY .....	141
FIGURE 5.8. LARGE SIZE OPERATING NETWORK OF VISHAKHAPATNAM CITY WITH CLUSTERS.....	142
FIGURE 5.9. TOTAL OPERATING COST AND TRAVEL DISTANCE FOR EXISTING AND PROPOSED ROUTE SET .....	146

# List of Tables

TABLE 2.1. VARIATION OF CONSTRAINTS IN BUS ROUTE NETWORK DESIGN.....	23
TABLE 2.2 - PARAMETERS FOR BUS ROUTE NETWORK DESIGN PROCEDURE.....	24
TABLE 2.3 - FACTORS DETERMINING DEMAND ESTIMATION FROM PASSENGER AND OPERATORS PERSPECTIVE .....	25
TABLE 2.4 - ROUTE CONSTRUCTION MODELS THROUGH EVOLUTIONARY ALGORITHMS .....	32
TABLE 3.1: POLLUTION COSTS FOR INDIA 2005.....	51
TABLE 3.2 - TRAVEL MODE USERS WHO ARE WILLING TO SHIFT FROM MODE TO MODE .....	61
TABLE 3.3 – TRAVEL CHARACTERISTICS OF PEOPLE WHO WANT TO SHIFT FROM 3-WHEELER AND SHARED 3-WHEELER TO BUSES.....	62
TABLE 3.4 - TRAVEL CHARACTERISTICS OF PEOPLE WHO WANT TO REMAIN USING 3-WHEELER MODE .....	62
TABLE 3.5 - TRAVEL CHARACTERISTICS OF PEOPLE WHO WANT TO SHIFT FROM BUSES TO 2-WHEELER.....	63
TABLE 3.6 - TRAVEL CHARACTERISTICS OF PEOPLE WHO WANT TO REMAIN USING BUS.....	63
TABLE 3.7 - WILLINGNESS TO SHIFT FROM 2-WHEELER TO BUS BASED ON INCOME GROUP .....	63
TABLE 3.8 - WILLINGNESS TO SHIFT FROM CARS TO BUS BASED ON INCOME GROUP .....	64
TABLE 3.9 - EXISTING TOTAL TRIPS AND FLOW FOR EACH MODE OF TRAVEL (NUMBERS) .....	64
TABLE 3.10: FINAL DEMAND; EXISTING AND POTENTIAL BUS TRANSIT DEMAND (NUMBERS).....	65
TABLE 3.11. VALUES OF ATT FOR TAZ AND PTTAZ SCHEMES (IN MINUTES) .....	80
TABLE 3.12 - ESTIMATED BUS STOP BASED ACCESSIBILITY INDEX FOR BUS TRANSIT OPERATION IN VISHAKHAPATNAM CITY.....	88
TABLE 4.1 - 5 NODE OD MATRIX (NUMBERS).....	94
TABLE 4.2 - ROUTE SELECTION IN ALL OR NOTHING ASSIGNMENT METHOD.....	94
TABLE 4.3 - A SAMPLE OF SHORTEST PATH ROUTES FOR INTEGER PARTITION PROCEDURE .....	96
TABLE 4.4 - RESULTS FROM K-SHORTEST PATH FOR SAMPLE 15 NODE NETWORK .....	103
TABLE 4.5 - ALL SHORTEST PATHS FOR ORIGIN 1 TO DESTINATION 14 IN SAMPLE 15 NODE NETWORK .....	104
TABLE 4.6 - FILTERED SHORTEST PATHS FOR ORIGIN 1 TO DESTINATION 5 BASED ON MINIMUM ALLOWABLE BUS STOPS IN SAMPLE 15 NODE NETWORK .....	104
TABLE 4.7 - FILTERED SHORTEST PATHS FOR ORIGIN 1 TO DESTINATION 8 BASED ON OVERLAPPING RULE IN SAMPLE 15 NODE NETWORK.....	105

TABLE 4.8 - RUNNING TIME FOR RGA AND RFA IN MATLAB .....	105
TABLE 4.9 - SET OF BEST ROUTES N_1 IN GA'S TMO .....	115
TABLE 4.10 - TRANSFERS IMPOSED TO SECOND BEST ROUTES N_2 IN GA'S TMO.....	115
TABLE 5.1 - ORIGIN AND DESTINATION MATRIX FOR MANDL'S SWISS NETWORK (NUMBERS) .....	125
TABLE 5.2 - PARAMETERS USED IN GENERATION AND OPTIMIZATION PHASES.....	126
TABLE 5.3. OPTIMUM SET OF ROUTES THROUGH PROPOSED ALGORITHM.....	127
TABLE 5.4. COMPARISON OF PROPOSED BRND MODEL WITH PREVIOUS RESULTS WITH 4 ROUTES.....	128
TABLE 5.5. COMPARISON OF PROPOSED BRND MODEL WITH PREVIOUS RESULTS WITH 6 ROUTES.....	129
TABLE 5.6. COMPARISON OF PROPOSED BRND MODEL WITH PREVIOUS RESULTS WITH 8 ROUTES.....	130
TABLE 5.7. VARIATION OF CONSTRAINTS IN BUS ROUTE NETWORK DESIGN .....	131
TABLE 5.8 - MODE WISE NUMBER OF VEHICLES IN VISHAKHAPATNAM CITY .....	135
TABLE 5.9- THE EXISTING SET OF ROUTES RUNNING THROUGHOUT THE URBAN AREA OF VISHAKHAPATNAM CITY.....	136
TABLE 5.10 - RUNNING COST OF OPERATION FOR VISHAKHAPATNAM CITY (2013) .....	139
TABLE 5.11 - FIXED AND MAINTENANCE COST OF OPERATION FOR VISHAKHAPATNAM CITY (2013).....	139
TABLE 5.12 - CHANGING EMISSION FACTORS IN GRAMS/VKM FOR FOUR MAJOR POLLUTANTS, VISHAKHAPATNAM, INDIA .....	140
TABLE 5.13 - PEAK TIME AND OFF PEAK TIME BUS FLOW IN VISHAKHAPATNAM CITY, INDIA (2013).....	140
TABLE 5.14 - DISAGGREGATE EMISSION FACTOR FOR BUS TRANSPORT PEAK TIME AND OFF PEAK TIME (G/KM).....	140
TABLE 5.15 - MARGINAL EXTERNAL COST OF BUSES IN VISHAKHAPATNAM (₹. VKM).....	140
TABLE 5.16 - SELECTED DESIGN AND DECISION PARAMETERS FOR LARGE TEST NETWORK .....	143
TABLE 5.17 - COMPARING BEST ROUTE SET OF PROPOSED MODEL WITH EXISTING OPERATION THROUGH QUALITY INDICATORS .....	145
TABLE 5.18 - COMPARING BEST ROUTE SET OF PROPOSED MODEL WITH EXISTING OPERATION THROUGH OPERATION COST.....	145
TABLE 5.19 - COST AND TRAVEL TIME IMPROVEMENT OF OPERATION FOR CLUSTERED AND NON-CLUSTERED OPERATING NETWORK.....	146
TABLE 5.20 - FLEET SIZE FOR PEAK AND OFF PEAK TIME FOR CLUSTERED AND NON-CLUSTERED OPERATING NETWORK.....	146
TABLE 5.20 – FINANCIAL VIABILITY RESULTS (COST IN RUPEES (₹)) .....	148

# Abbreviations

AC	Access Time
APSRTC	Andhra Pradesh State Road Transport Corporation
ASAI	Actual Stop Accessibility Index
AON	All or Nothing Assignment
ATT	Average Travel Time
BRND	Bus Route Network Design
BRND-FS	Simultaneous Bus Route Network Design and Frequency Setting
BT	Bus Transit
EBT	Experienced Based Technique
EG	Egress Time
EPDA	Exhaustive Passenger Demand Assignment Procedure
ESM	Exhaustive Search Methods
GA	Genetic Algorithm
GIS	Geographic Information System
GS	Greedy Search Algorithm
GVMC	Greater Visakhapatnam Municipal Corporation
HC	Hill Climbing
HHS	Household Survey
ISAI	Ideal Stop Accessibility Index
IPT	Intermediate Public Transport
NP-Hard	Nondeterministic Polynomial-Time Hard
OD	Origin and Destination
RFA	Route Filtration Algorithm
RGA	Route Generation Algorithm
PS	Particle Swarm Algorithm
PT	Public Transit
PTTAZ	Public Transport Traffic Analysis Zoning
SA	Simulated Annealing
SCRI	Stop Coverage Ratio Index

SMC	Small and Medium Sized City
SPS	Stated Preference Survey
TAZ	Traffic Analysis Zones
TMO	Transfer Minimization Operator
TS	Tabu Search
UAC	Uniform Annual Cost
VOT	Value of Time

# Notation

$(i)$	Origin bus stop (node)
$(j)$	Destination bus stop (node)

## Demand

$[d_{ij}^{bus}]$	Demand between origin and destination
$[pd_{ij}^{bus}]$	Potential demand between origin and destination
$[pd_{ij}^{2w}]$	2-wheeler potential demand to shift to bus for origin $[i]$ and destination $[j]$
$[pd_{ij}^{3w}]$	3-wheeler potential demand to shift to bus for origin $[i]$ and destination $[j]$
$[pd_{ij}^{bus\ to\ other}]$	Bus potential demand to shift to other modes for origin $[i]$ and destination $[j]$
$[pd_{ij}^{car}]$	Car potential demand to shift to bus for origin $[i]$ and destination $[j]$
$[pd^{bus}]$	Total potential demand
$[D_{ij}^{bus}]$	Total passenger demand between $[i]$ and $[j]$
$[dtr_{ij}]$	Number of transfer passenger demand between origin and destination

## TAZ delineation

$\left[ A_{z_{PTTAZ_m}}^{TAZ_n} \right]$	Trip attraction of proposed sub zone (PTTAZ) within particular TAZ
$\left[ P_{z_{PTTAZ_m}}^{TAZ_n} \right]$	Trip production of proposed sub zone (PTTAZ) within particular TAZ
$[d_{i,j}^{PTTAZ_{TAZ_n}}]$	PTTAZ total demand including existing and potential demand belonging to $n$ particular TAZ
$[pd_{i,j}^{pd,zone(n)}]$	Potential demand between origin $[i]$ and destination $[j]$ of TAZ where $[n]$ refers to zone
$[d_{i,j}^{ed,cell\ under\ network\ accessibility}]$	

	PTTAZ existing demand under accessibility coverage that aggregated to cell grid
$[d_{i,j}^{pd,all\ cells\ of\ TAZ_n}]$	PTTAZ potential demand under accessible and non-accessible area that aggregated to cell grid belonging to n particular TAZ
$[d_{i,j}^{zone\ (n)}]$	Existing demand between origin [i] and destination [j] of TAZ where [n] refers to zone number
$[Z'_{PTTAZ}]$	Set of proposed PTTAZ (Cell grids)
$[Z_{TAZ}]$	Set of existing TAZ
$[z'_{PTTAZ_m}]$	Sub-zone of proposed PTTAZ where [m] refers to cell grid (sub-zone) number
$[z_{TAZ_n}]$	Sub-zone of existing TAZ where [n] refers to zone number
$[\delta_{z'_{PTTAZ_m}}^{A,B,C,D} \ %]$	Population density ratio of each sub-section (A, B, C, and D) of cell grids (PTTAZ) where [m] refers to cell grid (sub-zone) number
$[\delta_{z'_{PTTAZ_m}}]$	Population density proportion of each cell grid PTTAZ

### Cost

$[TOC]$	Total cost of operation
$[C_{user}]$	User cost
$[C_{Pollution}]$	Pollution cost
$[PC_{\text{₹./km}}^{Pollution}]$	Pollution cost factor in rupees per km
$[C_{Operator}]$	Operation cost
$[C_{operator}^{Running}]$	Running costs of operation
$[C_{operator}^{Fixed}]$	Fixed costs of operation
$[C_{operator}^{Maintenance}]$	Maintenance costs of operation
$[C_{bus-km}^{Diesel}]$	Bus running cost per km
$[Cost\ of\ Tires_{\text{₹./km}}]$	Cost of bus tires in rupees per km
$[Depreciation_{\text{₹./km}}]$	Fixed cost of bus depreciation in rupees per km

[ <i>Driver cost</i> ₹./km]	Bus driver's salary in rupees per km
[ <i>Fuel price</i> ₹.per litre]	Cost of bus fuel in rupees per liter
[ <i>Motor vehicle taxes</i> ₹./km]	Motor vehicle tax as a part of operation cost in rupees per km
[ <i>Personnel wages</i> ₹./km]	Salaries of operating personnel in rupees per km
[ <i>Batteries and electrical</i> ₹./km]	Cost of bus batteries and electrical in rupees per km
[ <i>Regional, Zonal and Head of fice charges</i> ₹./km]	Charges of operator's offices in rupees per km
[ <i>Stores</i> ₹./km]	Bus operating store costs in rupees per km
[ <i>Workshops</i> ₹./km]	Cost of bus workshops in rupees per km
[ <i>Z</i> <sub>1</sub> ]	User related cost function
[ <i>Z</i> <sub>2</sub> ]	Operator related cost function
[ <i>Z</i> <sub>3</sub> ]	External pollution cost function
[ <i>W</i> ]	Weighting rate for objective function
[ <i>W</i> <sub>externality</sub> ]	External pollution cost related weightage in objective function
[ <i>W</i> <sub>operator</sub> ]	Operator related weightage in objective function
[ <i>W</i> <sub>user</sub> ]	User related weightage in objective function

## Time

[ <i>t</i> <sub>ij</sub> <sup>bus</sup> ]	In-vehicle travel time
[ <i>w</i> <sub>ij</sub> <sup>bus stop</sup> ]	Waiting time
[ <i>trp</i> <sub>ij</sub> <sup>Transfer</sup> ]	Transfer time
[ <i>ntrp</i> ]	Transfer penalty
[ <i>A</i> <sub>passenger</sub> <sup>bus stop</sup> ]	Access time
[ <i>E</i> <sub>passenger</sub> <sup>bus stop</sup> ]	Egress time
[ <i>T</i> <sub>total round trip Distance</sub> <sup>bus</sup> ]	Total round trip time by bus between origin and destination
[ <i>T</i> <sub>user</sub> ]	Passenger total travel time

[ $VOT$ ] Value of time

### Constraints

[ $n_k^{\text{Max number of bus stop}}$ ]	Maximum allowable number of bus stops serving through route [k]
[ $n_k^{\text{Min number of bus stop}}$ ]	Minimum allowable number of bus stops serving through route [k]
[ $l_k^{\text{Max number of links}}$ ]	Maximum allowable number of links serving through route [k]
[ $l_k^{\text{Min number of links}}$ ]	Minimum allowable number of links serving through route [k]
[ $f_k^{\text{max}}$ ]	Maximum allowable service frequency
[ $f_k^{\text{min}}$ ]	Minimum allowable service frequency
[ $F_{\text{max fleet size}}$ ]	Maximum allowable fleet size
[ $RL_k^{\text{1st shortest path}}$ ]	Route length of first shortest path
[ $RL_k^{\text{Max Route length}}$ ]	Maximum allowable route length
[ $RL_k^{\text{Min Route length}}$ ]	Minimum allowable route length
[ $\text{trnp}_{ij}^{\text{max Transfer}}$ ]	Maximum allowable number of transfers between origin [i] and destination [j]
[ $d_{un}$ ]	Unsatisfied demand
[ $F_{\text{route k}}$ ]	Required fleet size on route [k]
[C]	Capacity of transit vehicles
[F]	Available fleet size
[ $\alpha$ ]	Load factor
[ $\gamma_k$ ]	Route k that is below the minimum allowable number of bus stops

### Network design and frequency setting

[ $f_k^{\text{route}}$ ]	Service frequency on route [k]
[ $f_{\text{route k}}^{\text{off peak time}}$ ]	Service frequency on route [k] in off peak time operation
[ $f_{\text{route k}}^{\text{peak time}}$ ]	Service frequency on route [k] in peak time operation
[ $f_{ij}$ ]	Frequency between origin [i] and destination [j].

$[f_{ji}]$	Frequency between origin $[j]$ and destination $[i]$ .
$[f_{ij}^{route\ k}]$	Frequency between origin $[i]$ and destination $[j]$ of route $[k]$
$[Q_{\text{maximum load section}}]$	Maximum load section
$[trnp_{ij}^{route\ k}]$	Number of transfers between origin $[i]$ and destination $[j]$ of route $[k]$
$[trp_{ij,N,k}^{\text{Transfer}}]$	Number of transfers imposed for origin $[i]$ to destination $[j]$ in candidate route $[k]$
$[RL_k^{\text{Route length}}]$	Route $[k]$ length
$[R_{i,j,k}^{\text{candidate}}]$	Set of candidate routes $[k]$ for all origin $[i]$ and destination $[j]$
$[kSP^{\text{directed graph}}]$	Number of $k$ shortest path for directed graph
$[kSP^{\text{undirected graph}}]$	Number of $k$ shortest path for undirected graph
$[l_k^{\text{Route's number of links}}]$	Number of links serving through route $[k]$
$[wt_{tr}^{k,k'}]$	Waiting time at transfer point from route $[k]$ and exchange to second route $[k']$
$[\gamma]$	Direct and indirect demand satisfaction ratio
$[\gamma^{\text{direct ratio}}]$	Direct passenger demand satisfaction ratio
$[\gamma^{\text{indirect ratio}}]$	Indirect passenger demand satisfaction ratio
$[n_k^{\text{Route's number of bus stops}}]$	Number of bus stops serving through route $[k]$
$[RL]$	Trip route length
$[\Sigma d]$	Total operation satisfied passenger demand

### Genetic algorithm

$[dv_i]$	Substring $[i]$ in genetic algorithm coding
$[n_s^k]$	Node $[s]$ of particular $[k]$ route of population $[N]$ in genetic algorithm
$[l_i]$	Length of substring $[i]$ in genetic algorithm coding
$[\gamma_{Nm}^{\text{indirect ratio}}]$	Indirect passenger demand satisfaction ratio of population $[N]$ with size $[m]$ in genetic algorithm
$[N_m]$	Initial random population in genetic algorithm where $m$ is population size

$[N_m^k]$

Initial random population in genetic algorithm where k is number of individuals