

PREDICTION OF TWO-PHASE FREE SURFACE FLOW IN ROTATING CHANNELS

by

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CERTIFICATE

This is to certify that the thesis entitled **PREDICTION OF TWO-PHASE FREE SURFACE FLOW IN ROTATING CHANNELS** being submitted by **Veeraraghavan Ramanathan** to the **Indian Institute of Technology, New Delhi (India)** for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a bonafide research work carried out by him under my supervision and guidance. The research reports and the results presented in this thesis have not been submitted in parts or in full to any other University or Institute for the award of any degree or diploma.



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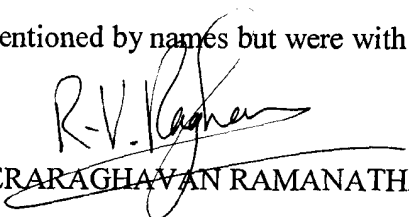
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ABSTRACT

Two-phase free surface flow in two-dimensional rotating channels is mathematically described using volume-averaged continuum equations, treating the solid and liquid phases as two interacting, coexisting continua. Governing continuity and momentum equations are written for each phase in a rotating reference. Turbulence is modeled *via* an eddy viscosity model. Transport of concentration is governed by the convection-diffusion equation. Special consideration is given to boundary conditions at the free surface.

The numerical algorithm involves three iterative steps, *viz.* computing (1) the mixture flow field and free surface by using a deforming mesh Galerkin finite element method, (2) the solid velocity at each grid point *via* a force balance, and (3) the concentration field using a convection-diffusion equation. These three steps are repeated until convergence. A combined Newton's iteration is used for the mixture flow field and the free surface. Parameter continuation (in particle diameter, Reynolds number and Rossby number) is used to generate sound initial guesses. Primitive variables (u, v, p) are used to compute the mixture velocity field. The kinematic condition is used to form the residual for the free surface. In some cases, upwinding is used to compute the concentration field. Velocity and concentration are interpolated biquadratically, while pressure is interpolated bilinearly. Free surface is interpolated using Hermite elements.

The numerical algorithm is first applied to compute single-phase free surface flow in rotating channels. Inviscid and viscous (both laminar and turbulent) solutions are obtained. The inviscid finite element and inviscid analytical results agree very well, partially validating the finite element code. Pohlhausen's integral analysis is used to solve the boundary layer equations obtained from scale arguments. The approximate boundary layer analysis results are within 50% of the finite element solutions, thus yielding a second partial validation of the finite element program developed in this research. Results of turbulent single-phase flow model agree well with the inviscid results outside the boundary layer.

Two-phase free surface flow is computed for a range of operating conditions typical for the coriolis wear tester channel. From the two-phase flow field, the wear along the channel base is calculated. The computed normalized wear rate

distributions show very good agreement with published experimental results of other authors.

Results indicate that a concentration rich layer is formed in all cases along the pressure side of the channel. However, the concentration reaches packing limit only for particles greater in diameter than a critical size. Furthermore, increased rotational speed of the channel, increased inlet concentration and increased particle diameter lead to increased concentration along the pressure side. Simultaneously, the concentration along the free surface decreases, tending to zero for large particle diameter. The wear rate along the channel length increases. Larger particles tend to cause greater wear rate (in the range studied). Predicted wear rates agree well with experimental results.

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