

A THESIS
ON
**DIFFERENCE METHODS FOR THE NUMERICAL
SOLUTION OF HYPERBOLIC SYSTEMS AND
SINGULAR PERTURBATION PROBLEMS**

by
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C E R T I F I C A T E

This is to certify that the thesis entitled 'Difference Methods for the Numerical Solution of Hyperbolic Systems and Singular Perturbation Problems', which is being submitted by A.C. Radhakrishna Pillai for the award of the degree, Doctor of Philosophy (Mathematics) , to the Indian Institute of Technology Delhi, is a bonafide record of research work done under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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