

COHERENCE PROPERTIES OF LASER
AND THERMAL RADIATIONS

by

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* This thesis is dedicated to the memory of my beloved *
* father, Late Dr. Shanti Sawrup Dudeja, who passed away *
* without giving me an opportunity to reciprocate even *
* an iota of his overflowing affection and deep attach - *
* ment to me. *
* *****

INTRODUCTION

Optical coherence theory is concerned with the statistical description of the fluctuations in the radiation, expressed in terms of correlation functions. The earliest investigations of coherence, limited to the second-order effects were carried out by Verdet¹, Michelson^{2,3}, van Cittert⁴, Zernike⁵, Hopkins⁶, Wolf⁷, and Mandel⁸ etc.⁹ Shortly after the second-order theory was formulated, some important experiments of Hanbury Brown and Twiss¹⁰ demonstrated that correlation can also be measured between quantities which depend quadratically on the field variables. This observation led to the study of higher-order correlation effects in optical fields.

An information about the statistics of an optical field is obtained mainly using photodetectors. Therefore it was necessary to find a relation between the statistics of photons in the field and the statistics of photoelectrons emitted by a photodetector exposed in this field. This was done by Mandel^{11,12}, who derived the so-called photodetection formula relating the probability density of time-integrated intensity to the distribution of photoelectrons. In 1963, Glauber^{13,14} developed the quantum theory of coherence based on quantum electrodynamics. Quantum correlation functions introduced by him represent the expectation values of normally ordered products of field operators and they are closely related to the quantities measured by means of

photodetectors. A number of problems in the quantum formulation of coherence properties of optical fields have also been solved by Haken¹⁵, Sargent, Scully and Lamb¹⁶, and Lax et al^{17,18}

The purpose of the present thesis is to study the phenomenon of coherence in laser and thermal radiations. We do not restrict ourselves to any particular theory out of those mentioned above, rather we make use of the theories proposed by Wolf, Mandel, Glauber, and Lax et al to study the statistical properties of optical fields. First, we describe the results of measurement of spatial coherence function in a laser, and use the correlation technique proposed by Wolf to study the polarization of light. We then proceed to confirm the van der Pol oscillator theory of lasers proposed by Lax by calculating the values of intensity correlation parameters. We use this theory to determine the phase of second-order coherence function for a laser. We next make use of the semiclassical theory proposed by Mandel to determine the probability density of light intensity from the statistical distribution of photoelectrons. This theory is then used to study the effect of temperature of the radiation field on photon-bunching. Finally, the basic statistical properties of optical fields have been studied with the help of diagonal coherent state representation of the density operator.

The thesis is divided into eight chapters.

In chapter I, we describe results of the measurement of the degree of spatial coherence across a He-Ne laser beam.¹⁹ We observe the visibility of the fringes produced on direct superposition of wave amplitudes from two space-points across the beam diameter with the help of optical fibers. It is observed that there is very little variation of the spatial coherence function across the beam diameter.

In chapter II, we study the degree of polarization of light by correlation technique²⁰. We suggest and give the relevant theory for two experiments for its measurement, one by measuring the amplitude correlations and the second by the measurement of intensity correlations. For the first experiment we have made use of the relation between degree of polarization of light wave and the elements of its coherency matrix.²¹ The second experimental set up can be used only for thermal light for which the elements of the intensity correlation matrix may be expressed in terms of the second-order moments of the electric-field components. For the first experiment, we have also, taken into account the change of the state of polarization of light as it passes through beam-dividers and reflectors. Finally, a comparison between the merits of the two experiments is discussed.

Chapter III serves as a background for the next two chapters of the thesis. In this chapter we review the main

features of the theory of lasers as developed by Lax et al^{17,18}. In particular, we study the fluctuation and coherence phenomenon in lasers operating below, at, and above their threshold of oscillation.

In chapter IV, we consider the coherence properties of light from a laser working near its threshold. The rotating wave van der Pol oscillator theory predicts a multiple exponential nature of the intensity correlation function, which has been qualitatively verified.²² We substantiate the qualitative agreement by numerically evaluating the values of different correlation parameters from the experimental curve and compare these with the theoretical values.²³ The agreement is found to be reasonable. This provides further confirmation of the rotating wave van der Pol oscillator model of the laser describing its mechanism in the threshold region.

Chapter V deals with the phase-problem in coherence theory. Gamo²⁴ proposed a three-point intensity correlation method for the determination of phase of the normalized second-order amplitude correlation function. Recently Cantrell et al²⁵ have done the calculations for a laser operating above, at, and below its threshold of oscillation. We make use of these two papers and suggest a method for the numerical evaluation of the second-order coherence function.²⁶ Our method is applicable for a laser working well below its threshold of oscillation.

In chapter VI, we discuss the determination of probability density of time-integrated light intensity from the statistical distribution of photoelectrons,²⁷ with the help of photo-counting formula of Mandel^{11,12}. Wolf et al²⁸ have calculated it with the help of Fourier inversion formula. In most of the cases, the inverse Fourier transform would be applicable equally with the inverse Laplace transform. However, there are certain cases of physical interest (discussed in the present chapter) where one cannot take the inverse Fourier transform of the function yet its inverse Laplace transform can still be taken. Therefore the method of inverse Laplace transform is used to determine the probability density of intensity. We have illustrated our method by taking examples of thermal, ideal laser, and the realistic model of laser light.

In chapter VII, we study the temperature dependence of photon-bunching in thermal radiation.²⁹ The results, obtained for the fluctuations in the number of photons by thermodynamic approach and by the photon-counting- formula approach of Mandel,³⁰ come out to be contradictory. However, if one takes into account the thermal fluctuations in the photodetector, the results from two different approaches agree. It is concluded that the high temperature of the radiation field is responsible for photon-bunching. The results are compared with those of Webb,³¹ Mandel³¹ and Abraham et al³².

In chapter VIII, we make use of the diagonal coherent state representation of density operator given by Glauber^{13,14} (or by Sudarshan³³) and propose yet another simple formulation³⁴ to calculate the characteristic function, probability of number of photons, and variance, in the case of coherent and Gaussian fields.

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