MICROSCALE FLOWS OVER PATTERNED TOPOGRAPHIES

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Dedicated to my parents. This would have not been possible without your patience and endless support.

Certificate

This is to certify that the thesis entitled **"Microscale flows over patterned topographies"** being submitted by **Mr. Mainendra Kumar Dewangan** to the Indian Institute of Technology, Delhi for the award of the degree of **Doctor of Philosophy** is a bonafide record of original research work carried out by him under my supervision in conformity with rules and regulations of the institute. The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any Degree or Diploma.

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Abstract

Topographical textures or surface corrugations are often employed in the discipline of microfluidics to achieve objectives such as mixing, modulation of wetting and frictional properties, and separations. Outside microfluidics, such textures form the basis of drag-reducing riblets engineered to turbulent flows and are natural features of fractured rocks carrying oil or water in geological formations.

Motivated by such applications, macroscopic characterization of creeping Newtonian flows establishing wetted contact with topographies is undertaken theoretically, using the effective slip length tensor as a metric for unconfined flows and the hydraulic permeability tensor as a metric for confined flows. Flows longitudinal as well as transverse to one-dimensional grooved/striped patterns are analyzed using spectral and asymptotic methods. Continuum scale fully resolved finite-element simulations are employed to assess the usefulness of asymptotic predictions. The main asymptotic approach used in this thesis treats the ratio of the characteristic size of the topographical pattern to the pattern pitch as a small parameter, the dimensionless amplitude.

First, for sinusoidally corrugated no-slip surfaces in shear flow, analytical models usable over a significantly wider amplitude range than those available from the literature are developed, taking advantages of symbolic computations, numerical-graphical convergence studies with Domb-Sykes plots and series-improvement techniques like Euler and Shanks transformation and Padé approximants.

Next, flows over corrugated surfaces with intrinsic slippage are studied. To understand the interplay between curvature and slippage, scaling laws are formulated by analyzing the singularity behaviour of transverse shear flows over perfectly slipping corrugations, a problem important in understanding the limits of superhydrophobic drag reduction. Based on these scaling laws, four different regimes in the interplay are delineated. The resultant effective-slip-length predictions for sinusoidally corrugated surfaces possess comparable simplicity but a wider amplitude-range and intrinsic-slip range of applicability

than those available from literature. A computer-extended regular perturbation theory is also developed, which allows exceptionally accurate predictions even at reasonably large values of the 'small (amplitude) parameter'.

Confined pressure-driven flows are studied in this thesis using a grid-free semi-analytical approach based on spectral analysis. This method has a faster (exponential) decay of errors compared to discretization-based methods such as finite-differences and finite-elements, and is also easily reducible to new and known analytical forms in various disparate limits. Further, the method brings out crucial physical characteristics of the flow that are inaccessible to asymptotic methods. For example, the permeability in longitudinal flow is shown to transition from faster-than-Poiseuille to slower-than-Poiseuille permeation as pattern pitch is reduced.

The final contribution of this thesis is a spectral-asymptotic approach to model confined flows over complex topographies specifiable by an arbitrary continuous function. Using a novel decomposition of the channel height effects into exponentially and algebraically decaying components, a simple surface-metrology-dependent relationship, which connects the eigenvalues of the effective permeability tensor, is obtained. Representative topographies assessed numerically include the infinitely-differentiable topography of a phase-modulated sinusoid with multiple local extrema and zero-crossings and the non-differentiable triangular-wave topography. Corners in triangular patterns and cusps in scalloped patterns are not found to be an impediment to meaningful and numerically accurate asymptotic predictions, contradicting an earlier suggestion from the literature. Several distinct applications of the theory to the friction-reduction and shear-stability performance of the recently developed lubricant impregnated patterned surfaces and scalloped and trapezoidal drag-reduction riblets are discussed, with comparison to experimental data from the literature for the last application.

सार

मिश्रण, गीलेपन और घर्षण गुणों के मॉड्यूलेशन और पृथक्करण जैसे उद्देश्यों को प्राप्त करने के लिए स्थलाकृतिक बनावट या सतही गलियारे को अक्सर माइक्रोफ्लुइडिक्स के अनुशासन में नियोजित किया जाता है। माइक्रोफ़्लुइडिक्स के बाहर, इस तरह के बनावट अशांत प्रवाह के लिए इंजीनियर खींच के आधारों का निर्माण करते हैं और भूवैज्ञानिक संरचनाओं में तेल या पानी ले जाने वाली खंडित चट्टानों की प्राकृतिक विशेषताएं हैं।

इस तरह के अनुप्रयोगों से प्रेरित, न्यूटन के प्रवाह को स्थलाकृतिक रूप से स्थलाकृतियों के साथ संपर्क स्थापित करने का मैक्रोस्कोपिक लक्षण वर्णन सैद्धांतिक रूप से किया जाता है, जो अपरिवर्तित प्रवाह के लिए एक मीट्रिक के रूप में प्रभावी स्लिप लंबाई टेंसर का उपयोग करता है और सीमित प्रवाह के लिए एक मीट्रिक के रूप में हाइड्रोलिक पारगम्यता पंप। अनुदैर्ध्य के साथ-साथ एक आयामी अंडाकार [धारीदार पैटर्न तक अनुप्रस्थ प्रवाह का विश्लेषण किया जाता है जो वर्णक्रमीय और असममित तरीकों का उपयोग करके किया जाता है। कॉन्टिनम स्केल ने पूरी तरह से हल किए गए परिमित-तत्व सिमुलेशन को असममित भविष्यवाणियों की उपयोगिता का आकलन करने के लिए नियोजित किया है। इस थीसिस में उपयोग किया जाने वाला मुख्य स्पर्शोन्मुख दृष्टिकोण पैटर्न पिच के लिए स्थलाकृतिक पैटर्न की विशेषता आकार के अनुपात को एक छोटे पैरामीटर, आयाम रहित आयाम के रूप में मानता है।

सबसे पहले, कतरनी प्रवाह में sinusoidally नालीदार नो-स्लिप सतहों के लिए, साहित्य से उपलब्ध उन लोगों की तुलना में काफी व्यापक आयाम रेंज पर प्रयोग करने योग्य विश्लेषणात्मक मॉडल विकसित होते हैं, जो सांकेतिक गणनाओं, संख्यात्मक-ग्राफिकल अभिसरण अध्ययनों का लाभ लेते हुए डोम्ब-साइक्स प्लॉट्स और श्रृंखला- यूलर और शैंक्स ट्रांसफॉर्मेशन और पेडे सन्निकट जैसी सुधार तकनीकें।

इसके बाद, आंतरिक फिसलन के साथ नालीदार सतहों पर प्रवाह का अध्ययन किया जाता है। वक्रता और फिसलन के बीच परस्पर क्रिया को समझने के लिए, पूरी तरह से फिसलने वाले गलियारों पर अनुप्रस्थ कतरनी प्रवाह की विलक्षणता का विश्लेषण करके स्केलिंग कानून तैयार किए जाते हैं, सुपरहाइड्रोफोबिक ड्रैग रिडक्शन की सीमाओं को समझने में महत्वपूर्ण समस्या। इन स्केलिंग कानूनों के आधार पर, इंटरप्ले में चार अलग-अलग नियम दिए गए हैं। Sinusoidally

नालीदार सतहों के लिए परिणामी प्रभावी-स्लिप-लंबाई भविष्यवाणियों में तुलनीय सरलता है, लेकिन साहित्य से उपलब्ध लोगों की तुलना में व्यापक आयाम-सीमा और प्रयोज्यता की आंतरिक सीमा है। एक कंप्यूटर-विस्तारित नियमित गड़बड़ी सिद्धांत भी विकसित किया गया है, जो 'छोटे (आयाम) पैरामीटर' के यथोचित बड़े मूल्यों पर भी असाधारण सटीक भविष्यवाणियों की अनुमति देता है।

वर्णक्रमीय विश्लेषण के आधार पर ग्रिड-मुक्त अर्ध-विश्लेषणात्मक दृष्टिकोण का उपयोग करके इस थीसिस में सीमित दबाव-संचालित प्रवाह का अध्ययन किया जाता है। इस पद्धति में विवेक-आधारित विधियों जैसे परिमित-अंतर और परिमितताओं की तुलना में त्रुटियों का तेज़ (घातांक) क्षय है, और विभिन्न विषम सीमाओं में नए और ज्ञात विश्लेषणात्मक रूपों के लिए आसानी से पुनर्वितरित किया जाता है। इसके अलावा, विधि प्रवाह की महत्वपूर्ण भौतिक विशेषताओं को सामने लाती है जो कि एसिम्प्टोटिक तरीकों के लिए दुर्गम हैं। उदाहरण के लिए, अनुदैर्ध्य प्रवाह में पारगम्यता को तेज-से-पॉइज़ुइल से धीमी-से-पॉइज़ुइल पारगमन में बदलने के लिए दिखाया गया है क्योंकि पैटर्न पिच कम हो गया है।

इस थीसिस का अंतिम योगदान एक मनमाना सतत कार्य द्वारा निर्दिष्ट जटिल स्थलाकृतियों पर सीमित प्रवाह के लिए मॉडल के लिए एक वर्णक्रमीय-विषम दृष्टिकोण है। चैनल ऊंचाई के प्रभावों के एक उपन्यास अपघटन का उपयोग घातीय और बीजगणितीय रूप से क्षय करने वाले घटकों में करते हुए, एक सरल सतह-मेट्रोलॉजी-निर्भर संबंध, जो प्रभावी पारगम्यता टेंसर के आइजनवेल्यूज को जोड़ता है, प्राप्त होता है। संख्यात्मक रूप से मूल्यांकन की गई प्रतिनिधि स्थलाकृतियों में कई स्थानीय विलुप्त होने और शून्य-क्रॉसिंग के साथ चरण-संशोधित साइनसॉइड की असीम-अलग-अलग स्थलाकृति और नॉनडिफ़रेन्सेबल त्रिकोणीय-लहर स्थलाकृति शामिल है। त्रिकोणीय पैटर्न में कॉर्न्स और स्कैलप्ड पैटर्न में क्यूप्स को सार्थक और संख्यात्मक रूप से सटीक विषम भविष्यवाणियों के लिए एक बाधा नहीं पाया जाता है, साहित्य से पहले के एक सुझाव का खंडन। अंतिम अनुप्रयोग के लिए साहित्य के प्रायोगिक डेटा की तुलना के साथ हाल ही में विकसित स्नेहक संसेचित नमूनों वाली सतहों के घर्षण-कटौती और कतरनी-स्थिरता के प्रदर्शन के सिद्धांत के कई अलग-अलग अनुप्रयोगों पर चर्चा की गई है।

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