

**KINEMATIC AND DYNAMIC ANALYSES OF A  
RAILWAY WHEELSET AS A SERIAL-CHAIN  
MECHANICAL SYSTEM**

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**DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

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*Submitted*

**in fulfilment of the requirements of the degree of Doctor of Philosophy**

*to the*



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**MAY 2023**

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## **Certificate**

This is to certify that the thesis entitled “**Kinematic and Dynamic Analyses of a Railway Wheelset as a Serial-Chain Mechanical System**” being submitted by **Mr. Rajeevlochana G. Chittawadigi** to the Department of Mechanical Engineering, Indian Institute of Technology Delhi for the award of degree of **Doctor of Philosophy** is a bonafide record of original research work carried out by him under my supervision in conformity with the rules and regulations of the institute.

The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any Degree or Diploma.

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We all enjoy going to different places or tourist destinations. Often, the destination alone is given the importance. However, the journey to the destination is equally rewarding where one learns many useful insights and encounters experiences. Striving hard to obtain a PhD degree is one such activity where the journey and the end-goal are equally important. I am very grateful to the almighty and everyone around me who helped me directly or indirectly to complete my research work and thesis at IIT Delhi. My journey has been very rewarding and so is the end-goal.

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Rajeevlochana G. Chittawadigi

## Abstract

Railway vehicles modelling has been an evolving field for more than a century. One of the important aspects in it is the wheel-rail interaction, i.e., understanding where contact happens between a moving wheelset and rails, and how the contact forces and moments are transmitted between them. Further, the wheelset interaction with the rails can be modelled as *Constrained* where contact is always ensured, and *Unconstrained* where the wheelset can penetrate into the rails or lift-off. The literature has methods for the detection of contact points that are analytical but approximate, and those based on numerical. The latter method is typically slower. In this thesis, a novel analytical method to represent a dicone wheelset with respect to knife-edge and cylindrical rails are proposed. It extends the concepts of Denavit-Hartenberg (DH) parameters that are generally used in closed-loop mechanisms or serial-chain mechanical systems such as open-loop robotic manipulators. Exact kinematic constraints for the wheelset to always have contact with the rails are proposed along with analytical expressions for the exact points of contact for both types of rails. Further, the wheelset is proposed as a serial-chain system like a robot manipulator by considering it as the last link or the end-effector of the manipulator. The contact forces (normal and creep) and moments, which are due to the wheel-rail interaction, were applied to the end-effector as external wrench. The forward dynamics analysis of a serial manipulator that uses DeNOC (Decoupled Natural Orthogonal Complement) was then applied to the *Constrained* approach for dicone on knife-edge, and cylindrical rails, and also for the *Unconstrained* approach of actual wheel-rail profiles. The kinematic and dynamic analyses of the wheelset were implemented in a software called RailAnalyzer. It allows users to input data and visualize the resulting motion of the wheelset in the form of 3-dimensional simulation in Autodesk Inventor CAD software with a provision to plot graphs.

## सार

रेलवे वाहन मॉडलिंग एक सदी से भी अधिक समय से एक उभरता हुआ क्षेत्र रहा है। इसमें महत्वपूर्ण पहलुओं में से एक व्हील-रेल इंटरैक्शन है, यानी यह समझना कि चलती व्हीलसेट और रेल के बीच संपर्क कहां होता है, और संपर्क बलों और टॉर्क: को उनके बीच कैसे प्रसारित किया जाता है। इसके अलावा, रेल के साथ व्हीलसेट इंटरैक्शन को “कंस्ट्रैन्टेड” (विवश शैली) के रूप में मॉडल किया जा सकता है जहां संपर्क हमेशा सुनिश्चित किया जाता है, और “अनकंस्ट्रैन्डेड” (अबाधित शैली) जहां व्हीलसेट रेल से “लिफ्ट-ऑफ” हो सकता है या प्रवेश कर सकता है। साहित्य में ऐसे संपर्क बिंदुओं का पता लगाने के तरीके हैं जो विश्लेषणात्मक लेकिन अनुमानित हैं, और जो संख्यात्मक पर आधारित हैं। बाद वाली विधि आमतौर पर धीमी होती है। इस थीसिस में “नाइफ-एज” और “सिलेंड्रिकल” रेल के संबंध में एक “डाइकोन” व्हीलसेट का प्रतिनिधित्व करने के लिए एक नई विश्लेषणात्मक विधि प्रस्तावित है। यह Denavit-Hartenberg (DH) पैरामीटर की अवधारणाओं को विस्तारित करता है जो आम तौर पर क्लोज्ड-लूप तंत्र या सीरियल-चेन मैकेनिकल सिस्टम जैसे ओपन-लूप रोबोटिक मैनिपुलेटर्स में उपयोग किए जाते हैं। दोनों प्रकार की रेलों के संपर्क के सटीक बिंदुओं के लिए विश्लेषणात्मक अभिव्यक्तियों के साथ, व्हीलसेट के साथ हमेशा संपर्क रखने के लिए सटीक “किनेमेटिक कांस्ट्रेंट्स” प्रस्तावित की जाती हैं। इसके अलावा, व्हीलसेट को रोबोट मैनिपुलेटर की तरह एक सीरियल-चेन सिस्टम के रूप में प्रस्तावित किया जाता है, इसे मैनिपुलेटर के अंतिम लिंक के रूप में माना जाता है। संपर्क बल (“नार्मल” और “क्रीप”) और टॉर्क:, जो व्हील-रेल इंटरैक्शन के कारण होते हैं, को “एक्सटर्नल रिंच” के रूप में अंतिम लिंक पर लागू किया गया था। एक सीरियल मैनिपुलेटर का फॉरवर्ड डायनेमिक्स विश्लेषण जो DeNOC (Decoupled Natural Orthogonal Complement) का उपयोग करता है, तब नाइफ-एज, और सिलेंड्रिकल रेल पर डाइकोन के लिए “कंस्ट्रैन्टेड एप्रोच” और वास्तविक व्हील-रेल प्रोफाइल के “अनकंस्ट्रैन्डेड एप्रोच” के लिए भी लागू किया गया था। RailAnalyzer नामक सॉफ्टवेयर में व्हीलसेट के कीनेमेटिक और डायनेमिक विश्लेषणों को लागू किया गया था। यह उपयोगकर्ताओं को डेटा इनपुट करने की अनुमति देता है और ग्राफ़ को प्लॉट करने के प्रावधान के साथ Autodesk Inventor CAD सॉफ्टवेयर में 3-आयामी सिमुलेशन के रूप में व्हीलसेट की परिणामी गति की कल्पना करता है।

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## List of Symbols and Abbreviations

The symbols have been described at the places of their first occurrence. Notational rules followed for symbols in this thesis are given below:

- Non-bold, lower-case and italicized letters refer to scalars.
- Non-bold, upper-case and italicized letters refer to points, curves and axes of coordinate frames.
- Bold and lower-case letters refer to vectors.
- Bold and upper-case letters refer to matrices.

### Abbreviations

API: Application Programming Interface

BG: Broad Gauge

CAD: Computer Aided Design

DEM: Discrete Element Modeling

DeNOC: Decoupled Natural Orthogonal Complement

EL: Euler-Lagrange

DH: Denavit-Hartenberg

DOF: Degree(s)-of-freedom

ELM: Enhanced Levin's Method

GIM: Generalized Inertia Matrix

GUI: Graphical User Interface

HTM: Homogenous Transformation Matrix

LHB: Linke-Hofmann-Busch

MCI: Matrix of Convective Inertia

MBD: Multi-body Dynamics

MDH: Modified Denavit-Hartenberg

MVC: Model-View-Controller

NE: Newton-Euler

NOC: Natural Orthogonal Complement

QSIC: Quadric Surface Intersection Curve

SDH: Six DH

VCI: Vector of Convective Inertia

WRI: Wheel-rail Interaction