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A THESIS ON

EFFECTS OF SLIP, SUCTION, ROTATION AND MAGNETIC FIELD ON FLOW AND HEAT TRANSFER PROBLEMS

BY

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C E R T I F I C A T E

This is to certify that the thesis entitled 'Effects of slip, suction, rotation and magnetic field on flow and heat transfer problems' that is being submitted by Mrs. S. Gulati for the award of the Degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by her under my supervision and guidance. Mrs. Gulati has worked for the last two years in the Department of Mathematics, Indian Institute of Technology, Delhi and the thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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S Y N O P S I S

The importance of the effects of the phenomena of slip, suction, rotation and magnetic field on the flow and heat transfer problems in hydrodynamics is well-known.

In the rarefied medium, the velocity of the gas in the immediate vicinity of the body does not correspond to the velocity of the body and hence the slip effects have to be taken into account. It is open to question as to whether Navier-Stokes equations are valid at low densities; however there are enormous ambiguities connected with the application of Burnett, Thirteen Moment and other equations. Hence in the present work, our concern is mainly with situations where the deviations from the continuum behaviour are very small so that Navier-Stokes equations may be used. Again, the effects of suction are important in controlling the boundary layer and thus avoiding separation which further results in the reduction of pressure drag acting on the body. It also causes delay in transition from the laminar to turbulent flow. For the case of an infinite liquid rotating as a rigid body about an axis, the amount of energy possessed by it is infinite and it is of interest to know how small disturbances propagate in such a liquid. Further, the interaction of flow field with the magnetic field for a conducting viscous fluid makes a no less significant study. It is well known that when a conductor moves in a magnetic field, electric currents are induced in it. These currents experience a mechanical force (called the

Lorentz force) due to the presence of the magnetic field which tends to modify the initial motion of the conductor. Moreover the induced currents generate their own magnetic field which is added to the primitive magnetic field. Thus there is an inter-locking between the motion of the conductor and the electromagnetic field.

The investigation of these effects in the present thesis comprises of eight chapters. Chapter I consists of the general introduction to the importance and application of these effects in detail. A short survey of the problems discussed and their relevant literature follows. The basic equations for the flow and heat transfer phenomena and for the electromagnetic quantities as employed in different situations in the following chapters have been listed for ready reference.

Chapter II deals with the hydromagnetic flow between two non-conducting flat plates executing steady state linear oscillations in their own planes under a uniform transverse magnetic field. A significant point is that the two plates oscillate with any phase difference, same frequency and different amplitudes. Two cases have been considered (i) when the magnetic field is fixed relative to the fluid and (ii) when the magnetic field is fixed relative to the plate. In both the cases it is found that as the Reynolds number tends to zero, the shearing stress on the two plates tend to be equal and when it tends to infinity the shearing stress on the lower plate behaves as if the other plate were absent. However in the latter case, if the upper plate were stationary, the shear stress on it in case (i) tends to zero but in case (ii) it is not so. The effect of the magnetic field on the skin friction coefficients for various values of the phase difference at the end of a time period in the two cases has also been investigated. In case (ii) it first increases, attains a maximum and then starts decreasing in all situations at both the plates with an increase in the magnetic parameter; where as in case (i) except for the case when the plates oscillate in opposite directions, the skin friction coefficients steadily increase with an increase in the magnetic parameter. For this exceptional case (out of the few cases considered for illustration), it increases at the lower plate but decreases at the upper one with an increase in the magnetic parameter. Viscous and magnetic boundary layers are also seen to develop at both the plates in the two cases as the Reynolds number increases.

Chapter III consists of the steady slow (inertialess) laminar flow and heat transfer of a viscous fluid through a converging channel and a tapered channel with small suction or injection (so small that the conditions of slow motion are not violated) at the walls, varying as any power (except for the case when it is inversely proportional) of the radial distance from the virtual line of intersection. A constant volume flux is maintained at the converging end. As the inertia terms have been neglected the equations of motion become linear and hence the method of superposition becomes applicable. Introducing a stream function so as to satisfy the equation of continuity and eliminating the pressure term from the equations of momentum we get a biharmonic equation which is solved by assuming a suitable form for the stream function. Next, the heat energy equation is solved for the inertialess flow with only small uniform suction or injection. For discussion purposes, two cases have been exhaustively studied (a) when there is uniform suction at one wall and equal injection at the other and (b) when there is equal uniform suction at both the walls. Since in practice, all channels do have a section of finite width, it is interesting to compare the results for a tapered channel to those of a converging channel. We observe that in case (a) all the results for both the geometries are the same while in case (b) they differ. The radial pressure drop in case (a) is always increased by suction except for the central plane where it does not contribute. In case (b) for tapered channel suction increases, does not affect or decreases it depending on the radial distance; but for the converging channel suction always decreases it. However the radial pressure drop for the tapered channel is always greater than that for the converging channel in case (b). Again the shear stress in case (a) is always decreased by suction but in case (b) for tapered channel the effect of suction on the shear stress varies depending on the radial distance while for the converging channel suction increases it at one wall but decreases it on the other. Along the central plane, however, suction does not contribute for either of the two geometries. Similar analyses for the normal stress difference, skin friction coefficients at the walls, temperature distribution and rate of heat transfer at the walls make a very interesting study.

In chapter IV a similar problem as discussed in chapter III has been considered for a conical tube (duct) and a tapered tube with suction at the wall varying as any power of the radial distance from the virtual vertex. The method of solution is similar to that discussed in the previous chapter. For discussion purposes, the case of uniform suction has been widely discussed and results for the conical duct and the tapered tube have been compared. It is found that the radial pressure drop for the former is always decreased by suction where as for the latter, suction increases, does not affect or decreases it depending on the radial distance. Again the shear stress for the conical duct is always decreased by suction for all values of the radial distance but when we go from the axis towards the wall in any fixed section of it, the effect of suction is to damp its rate of increase. For the tapered tube, again, the effects of suction depend on the radial distance. However, the shear stress for the tapered tube is always greater than that for the conical duct. Suction also helps to increase the temperature distribution and the rate of heat transfer at the wall for both the geometries the increase being more for the tapered tube. Transverse pressure drop, normal stress differences and skin friction coefficients are a few more other results which have also been discussed.

The flow problem for the rotatory oscillations of a sphere experiencing normal suction or injection in an infinite mass of rotating incompressible viscous fluid forms the subject of study in chapter V. Assuming that small suction or injection is present and that the amplitude of oscillations of the sphere is small, the convective terms in the momentum equations have been neglected. Further assuming the rotation parameter (the reciprocal of Rossby number) to be small, the velocity and pressure are perturbed in terms of it and the solution upto the first order of approximation has been computed. The effect of suction or injection is to produce a drag on the sphere which is unaffected by rotation. It is also interesting to note that suction or injection alone generates only the azimuthal component of vorticity, as rotation alone does but in the presence of suction or injection, all the components of vorticity are generated. Further the couple acting on the sphere has been evaluated and the variations of its factors of inertia force and frictional force with varying viscosity and rotation parameters have been discussed. It is found that with an increase in the viscosity parameter for a fixed rotation parameter, both the factors decrease where as for a fixed viscosity parameter and increasing rotation parameter, both increase.

Chapter VI studies the small steady rotation of a spheroid having a small ellipticity in an infinite mass of incompressible viscous conducting fluid, allowing slip at its surface under a uniform transverse magnetic field acting along the axis of rotation. Neglecting the inertia terms for slow flow, the physical quantities are perturbed in terms of the magnetic Reynolds number assuming the magnetic pressure number to be of order unity. The pressure, velocity field and the magnetic field are evaluated upto first order of approximation. Finding the expression for the couple acting on the spheroid, the effects of varying the magnetic Reynolds number, the slip, the viscosity parameter and the ellipticity on it have been studied. It is seen that the effect of slip is to decrease the couple where as the magnetic field tends to increase it. In the presence of slip, the effect of the magnetic field is seen to diminish. A small deformation of the type considered also decreases it.

In chapter VII, a similar problem has been investigated as in the previous chapter except that the spheroid is now executing rotatory oscillations instead of steady rotation. The technique of solution is the same. Variations in the factors of inertia force and the frictional force comprising the couple acting on the spheroid have been studied varying the various parameters involved.

In the last chapter, the shear flow of a finitely electrically conducting elastico-viscous fluid (Walters liquid B") past a porous flat plate in the presence of a constant pressure gradient and a uniform transverse magnetic field has been investigated. This study has been divided into two stages; first the viscous case solution is obtained and then the elastico-viscous solution is found, assuming that it is a perturbation of the viscous case solution. It is interesting to note that in the presence of a pressure gradient, the velocity and the magnetic field do interact at far distance. The electric current density vector at far distance is found to vary as the constant pressure gradient and the electric field is found to be constant throughout the flow field. Elasticity of the fluid does not contribute to both of these quantities at far distance. The shear stress at the plate too remains unaffected by elasticity.

C O N T E N T S

Chapter		Pages
I	GENERAL INTRODUCTION	1 - 13
	1.1 Introduction	1 - 5
	1.2 Problems investigated	5 - 9
	1.3 Basic equations	9 - 13
II	HYDROMAGNETIC FLOW BETWEEN TWO OSCILLATING FLAT PLATES	14 - 34
	2.1 Introduction	14 - 14
	2.2 Formulation and solution	14 - 20
	2.3 Discussion	20 - 29
III	STEADY SLOW LAMINAR FLOW AND HEAT TRANSFER OF A VISCOUS FLUID THROUGH A TAPERED CHANNEL WITH SUCTION AND INJECTION	35 - 59
	3.1 Introduction	35 - 36
	3.2 Formulation and solution of the flow problem	36 - 41
	3.3 Formulation and solution of the heat transfer problem	41 - 43
	3.4 Discussion	43 - 54
IV	STEADY SLOW LAMINAR FLOW AND HEAT TRANSFER OF A VISCOUS FLUID THROUGH A TAPERED TUBE WITH SUCTION OR INJECTION	60 - 82
	4.1 Introduction	60 - 60
	4.2 Formulation and solution of the flow problem	60 - 65

C O N T E N T S

Chapter		Pages
	4.3 Formulation and solution of the heat transfer problem	65 - 67
	4.4 Discussion	68 - 77
V	ROTATORY OSCILLATIONS OF A SPHERE WITH VARIABLE SUCTION OR INJECTION IN A ROTATING VISCOUS FLUID	83 - 97
	5.1 Introduction	83 - 83
	5.2 Formulation and solution	83 - 89
	5.3 Discussion	89 - 96
VI	SLOW STEADY ROTATION OF A SPHEROID WITH SLIP IN A CONDUCTING FLUID	98 - 113
	6.1 Introduction	98 - 99
	6.2 Formulation	99 - 103
	6.3 Solution	103 - 112
	6.4 Discussion	112 - 112
VII	ROTATORY OSCILLATIONS OF A SPHEROID WITH SLIP IN A CONDUCTING FLUID	114 - 135
	7.1 Introduction	114 - 114
	7.2 Formulation	114 - 117
	7.3 Solution	117 - 127
	7.4 Discussion	127 - 133

C O N T E N T S

Chapter		Pages
VIII	SHEAR FLOW OF A FINITELY ELECTRICALLY CONDUCTING ELASTICO-VISCOUS FLUID PAST A POROUS FLAT PLATE IN THE PRESENCE OF A CONSTANT PRESSURE GRADIENT AND A UNIFORM TRANSVERSE MAGNETIC FIELD	136 - 148
	8.1 Introduction	136 - 137
	8.2 Formulation	137 - 140
	8.3 Viscous case solution	140 - 142
	8.4 Elastico-viscous case solution	142 - 147
	8.5 Discussion	147 - 148
	BIBLIOGRAPHY	149 - 151