ON SOME PROBLEMS OF ANALYSIS OF THIN NONCIRCULAR OPEN CYLINDRICAL SHELLS

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K. SURESH

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CERTIFICATE

This is to certify that the thesis entitled "On Some Problems of Analysis of Thin Noncircular Open Cylindrical Shells" being submitted by Mr. K. Suresh to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him. Mr Suresh has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted, in part or full, to any other University or Institute for the award of any degree or diploma.

P.K. Bhattacharyya

(P.K. Bhattacharyya)
Assistant Professor
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110016.

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ON SOME PROBLEMS OF ANALYSIS OF THIN NONCIRCULAR CYLINDRICAL SHELLS

SYNOPSIS

The dissertation entitled "On Some Problems of Analysis of Thin Noncircular Open Cylindrical Shells" deals with the solution of problems of open thin elastic noncircular cylindrical shells based on Kirchhoff-Love hypotheses. From the point of view of importance and usefulness in practical applications two classes of problems have been studied. Hence the dissertation is divided into two parts. Part I deals with an 'exact' (or 'classical') method of analytic solution of class $\mathcal{K}$ of problems of shells of constant thickness t_0 with their curvilinear edges $\breve{\Gamma}_1, \breve{\Gamma}_2$ supported by diaphragms which are infinitely rigid in their planes but perfectly flexible in the direction normal to their planes. Then the boundary conditions along $\breve{\Gamma}_1, \breve{\Gamma}_2$ are

$$T_1\Big|_{\breve{\Gamma}_\alpha} = M_1\Big|_{\breve{\Gamma}_\alpha} = v\Big|_{\breve{\Gamma}_\alpha} = w\Big|_{\breve{\Gamma}_\alpha} = 0 \quad (\alpha = 1,2) \qquad (1)$$

The straight edges $\bar{\Gamma}_1, \bar{\Gamma}_2$ are subjected to arbitrary physically meaningful boundary conditions (see equations ((17)). Part II is devoted to the variational methods (classical and finite element methods) of solutions of problems of 'shallow' (in the generalised sense) cylindrical shells rigidly clamped along the boundary $\Gamma = \bigcup_{\alpha=1}^{2} (\breve{\Gamma}_\alpha \cup \bar{\Gamma}_\alpha)$ such that the boundary conditions along the edge Γ are given by:

$$u\Big|_{\Gamma} = v\Big|_{\Gamma} = w\Big|_{\Gamma} = \frac{\partial w}{\partial n}\Big|_{\Gamma} = 0 \qquad (2)$$

The general introduction includes the survey of the existing literature and the significance of the problem dealt with in the dissertation.

Chapter 1 deals with the study of the differential geometry of the shell middle surfaces in tensor form, geometry of different profiles of the middle surface and polynomial approximation to the curvature of the profile. The middle surface of the cylindrical shell in $\mathbb{R}^3$, which is a developable one, is in one-to-one correspondence with

$$\Omega = \{(x,s) : 0 < x < l, -s_0 < s < s_0\} \subset \mathbb{R}^2$$

which in turn is transformed into

$$\Omega_0 = \{(\xi,\eta) : 0 < \xi < l/R_0, -\eta_0 < \eta < \eta_0\} \subset \mathbb{R}^2 \qquad (3)$$

by introducing dimensionless variables $\xi = x/R_0$, $\eta = s/R_0$, where l is the length of the shell, R_0 is the radius of curvature of the profile at $s = 0$ such that the radius of curvature R at any point η is given by $R(\eta) = R_0\,\rho(\eta)$, ρ being the transformed radius of curvature. For a class of 'regular', 'symmetric' profiles (plane curves), expressions for the transformed curvature $1/\rho(\eta)$ and its derivatives can be developed in the form of power series uniformly and absolutely convergent in $|\eta| < \eta_{00}$, i.e.

$$1/\rho(\eta) = \sum_{k=0}^{\infty} a_k \eta^k \quad |\eta| < \eta_{00} \; (\eta_{00} > 0) \qquad (4)$$

But in many cases the algorithm for such an expression is associated with formidable computational difficulties. Hence Bernstein polynomial approximations for $1/\rho$ and its derivatives have been developed as follows: $\forall n \in \mathbb{N}$

$$\frac{1}{\rho} \approx B_n\left(\frac{1}{\rho}\right) = \frac{1}{\eta_0^n} \sum_{k=0}^{n} \binom{n}{k} \frac{1}{\rho}\left(\frac{k\eta_0}{n}\right) \eta^k (\eta_0 - \eta)^{n-k}$$

$$= \sum_{k=0}^{n} d_k \eta^k \qquad (5)$$

uniform convergence of the approximation (5) and its derivatives has been studied.

Part I Consists of Chapters 2, 3 and 4

Chapter 2 deals with the Generalised Moment Theory in the complex form of Novozhilov, since the system of governing equations of open cylindrical shells in real form is extremely complex in nature and cannot be reduced to a single equation of higher order. In the complex form of Novozhilov the system of governing equations of order eight is reduced to a single linear partial differential equation

$$\mathcal{L}(\widetilde{T}) \equiv \Delta[\rho\Delta(\widetilde{T})] + \frac{\partial}{\partial\eta}\left(\frac{1}{\rho}\frac{\partial\widetilde{T}}{\partial\eta}\right) + i\mu_1^2\ \frac{\partial^2\widetilde{T}}{\partial\xi^2} = \widetilde{f} \quad (6)$$

of order four for the unknown auxiliary complex force function $\widetilde{T} : \Omega_0 \to \mathcal{C}$, where

$$\Delta(\cdot) = \frac{\partial^2(\cdot)}{\partial\xi^2} + \frac{\partial^2(\cdot)}{\partial\eta^2},\ \mu_1^2 = \sqrt{12(1-\nu^2)}\ R_0/t_0,$$

$\widetilde{f} : \Omega_0 \to \mathcal{C}$ is a known function dependent on the components q_i of the surface load vector $\vec{q} = (q_1, q_2, q_3)$ and μ_1. For $f \in \mathcal{C}(\Omega_0) \cap D^{(1)}(\Omega_0)$, the unknown function $\widetilde{T} \in \mathcal{C}^{(4)}(\Omega_0) \cap D^{(5)}(\Omega_0)$ and for fixed $\eta \in (-\eta_0, \eta_0)$, $\widetilde{f}_\eta : \xi \in (0, l/R_0) \to \widetilde{f}_\eta(\xi) = \widetilde{f}(\xi, \eta) \in \mathbb{C}$ belongs to $\mathcal{C}(0, l/R_0) \cap D^{(1)}(0, l/R_0)$, $\widetilde{T}_\eta : \xi \in (0, l/R_0) \to \widetilde{T}_\eta(\xi) = \widetilde{T}(\xi, \eta) \in \mathbb{C}$ belongs to $\mathcal{C}^{(4)}(0, l/R_0) \cap D^{(5)}(0, l/R_0)$ (7)
Taking the boundary conditions (1) into account, $\widetilde{T}_\eta$ and $\widetilde{f}_\eta$ are given the following Fourier representations:

$$\widetilde{T}(\xi, \eta) = \widetilde{T}_\eta(\xi) = \sum_{n=1}^{\infty} \widetilde{T}_n(\eta) \sin \lambda_n \xi;$$

$$\tilde{f}(\xi,\eta) = \tilde{f}_\eta(\xi)$$

$$= \sum_{n=1}^{\infty} \tilde{f}_n(\eta) \sin \lambda_n \xi \qquad (8)$$

Since $[\sin \lambda_n \xi]_{n=1}^{\infty}$ $(\lambda_n = n\pi R_o/l)$ is a closed and complete orthogonal system in $L^2(0, l/R_o)$. It has been proved that for $\tilde{T} \in \mathcal{E}^{(4)}(\Omega) \cap D^{(5)}(\Omega)$, the Fourier series (8) can be termwise successively differentiated four times with respect to both ξ and η. Substituting (8) in (6) an infinite system of linear ordinary differential equations is obtained: $\forall n \in \mathbb{N}$

$$\begin{aligned} L_n(\tilde{T}_n) \equiv{} & \rho \tilde{T}_n^{(4)} + 2\rho^{(1)} \tilde{T}_n^{(3)} + [\rho^{(2)} - 2\lambda_n^2 \rho + 1/\rho]\, \tilde{T}_n^{(2)} \\ & + [-2\lambda_n^2 \rho^{(1)} + (1/\rho)^{(1)}]\, \tilde{T}_n^{(1)} + [\lambda_n^4 \rho - \lambda_n^2\, \rho^{(2)} - \\ & - i\mu_1^2 \lambda_n^2]\tilde{T}_n = \tilde{f}_n \qquad (9) \end{aligned}$$

where $\tilde{f}_n$ is a known function of η $\forall n \in \mathbb{N}$; ρ, $1/\rho$ are of the form (4) or (5). Then the corresponding homogeneous equation:

$$L_n(\tilde{T}_n) = 0, \qquad (10)$$

whose coefficients are analytic in $|\eta| \leq \eta_o < \eta_{oo}$, has four linearly independent analytic solutions of the form

$$\underset{r}{\tilde{T}_n} = \eta^r \sum_{k=0}^{\infty} \underset{r}{g_k}\, \eta^k \quad |\eta| \leq \eta_o < \eta_{oo} \qquad (11)$$

where $r = 0,1,2,3$ and $\underset{r}{g_k}$'s are known complex coefficients and these are determined using Cauchy-Frobenius method. Since

$\forall n \in \mathbb{N}$, $\tilde{f}_n \in \mathscr{L}(-\eta_0,\eta_0) \cap D^{(1)}(-\eta_0,\eta_0)$, a particular solution $\tilde{T}_{n,p}$ of equation (9) is constructed in the form:

$$\forall n \in \mathbb{N},\quad \tilde{T}_{n,p} = \int_0^{\eta} \tilde{\varphi}_n(\eta-\tau)\ \frac{\tilde{f}_n(\tau)}{\rho(\tau)}\ d\tau \qquad (12)$$

where $\tilde{\varphi}_n$ is the solution of the auxiliary initial value problem of the homogeneous equation (10) with the initial conditions $\tilde{\varphi}_n(0) = \tilde{\varphi}_n^{(1)}(0) = \tilde{\varphi}_n^{(2)}(0) = 0$, $\tilde{\varphi}_n^{(3)}(0) = 1$. If $\tilde{f}_n$ in (9) is analytic in $|\eta| < \eta_0$, an analytic particular solution of (9) exists and is constructed in the form

$$\forall n \in \mathbb{N},\quad \tilde{T}_{n,p} = \sum_{k=4}^{\infty} A_k \eta^k \qquad |\eta| \leq \eta_0 \qquad (13)$$

where A_k's are complex coefficients. Then the general solution of equation (8) has the form:

$$\forall n \in \mathbb{N},\quad \tilde{T}_n(\eta) = \sum_{r=0}^{3} \tilde{C}_r \tilde{T}_{n_r} + \tilde{T}_{n,p}(\eta) \qquad |\eta| \leq \eta_0 \qquad (14)$$

such that $\tilde{T}$ is given by

$$\tilde{T}(\xi,\eta) = \sum_{n=1}^{\infty} \tilde{T}_n(\eta) \sin \lambda_n \xi \qquad |\eta| < \eta_0 \qquad (15)$$

where $\tilde{C}_r = C_{1r} + iC_{2r}$; C_{1r}, $C_{2r} \in \mathbb{R}$ are arbitrary constants of integration which are determined from the boundary conditions (17) along the edges $\bar{\bar{\Gamma}}_{\alpha}$. The stresses, moments and displacements in the shell are derived from $\tilde{T}(\xi,\eta)$.

For a large class of problems encountered in practical applications, in which an approximate membrane stress condition exists in the interior of the shell, it is permissible in theory of shells to accept the solution of the nonhomogeneous membrane problem as a particular solution of the given nonhomogeneous generalised moment problem. This method of construction of a particular solution has been used in the numerical experiments.

Chapter 3 deals with the 'Generalised Moment Theory', (also called the 'Best-First Order Approximation Theory') of Sanders in complex form, the system of governing equations of which can be put in tensor form (unlike Novozhilov's equations in complex form) as follows:

$$P^{\alpha\beta}_{,\beta} + \frac{1}{\mu} b^{\alpha}_{\gamma} Q^{\gamma\beta}_{,\beta} - \frac{1}{2\mu} e^{\alpha\beta} (B_{\gamma\delta} Q^{\gamma\delta})_{,\beta} + q^{\alpha} = 0$$

$$Q^{\alpha\beta}_{,\alpha\beta} - \mu b_{\alpha\beta} P^{\alpha\beta} + q^{3} = 0 \qquad (16)$$

where $P^{\alpha\beta}$ and $Q^{\alpha\beta}$ define auxiliary complex tensor fields, $e^{\alpha\beta}$ is the permutation tensor, $B^{\alpha\beta}$ is a known tensor defined by the curvature tensor $b^{\alpha\beta}$, $\vec{q} = (q^1, q^2, q^3)$ is the surface load vector. For the boundary conditions (1) on the curvilinear edges $\check{\Gamma}_1, \check{\Gamma}_2$, the method of solution developed in Chapter 2 is extended to the present case with necessary modifications and changes.

For various 'classical' and nonclassical boundary conditions along the straight edges $\bar{\Gamma}_1, \bar{\Gamma}_2$ of cylindrical shells, which are frequently met with in practical applications; for example $\forall \alpha = 1, 2$

Free edges: $T_2|_{\bar{\Gamma}_\alpha} = 0, \quad S|_{\bar{\Gamma}_\alpha} = 0, \quad M_2|_{\bar{\Gamma}_\alpha} = 0$

$$Q_2^*|_{\bar{\Gamma}_\alpha} = \left(Q_2 + \frac{1}{R_0}\frac{\partial H}{\partial \xi}\right)\Big|_{\bar{\Gamma}_\alpha} = 0 \qquad (17a)$$

Hinged with fixed support:

$$M_2|_{\bar{\Gamma}_\alpha} = 0, \quad u|_{\bar{\Gamma}_\alpha} = 0, \quad v|_{\bar{\Gamma}_\alpha} = 0, \quad w|_{\bar{\Gamma}_\alpha} = 0 \qquad (17b)$$

Rigidly clamped edge:

$$u|_{\bar{\Gamma}_\alpha} = 0, \quad v|_{\bar{\Gamma}_\alpha} = 0, \quad w|_{\bar{\Gamma}_\alpha} = 0, \quad \frac{\partial w}{\partial n}\Big|_{\bar{\Gamma}_\alpha} = 0 \qquad (17c)$$

Edge stiffned by beams:

Shell over many spans:

(whose boundary conditions are very complex in nature), extensive numerical experiments have been carried out utilising the algorithms of solution developed earlier and the results are presented at the end of Chapters 2 and 3. The results obtained conclusively establish the authenticity of the method of solution developed for these problems.

Chapter 4 is devoted to : (a) the comparative study of the numerical results obtained in chapters 2 and 3 based on the 'exact' theories of Novozhilov and Sanders (valid for shells of arbitrary length), (b) comparison of the numerical results based on 'exact' theories with those based on the 'approximate' theories of Donnell and Vlasov (valid for shells of short and intermediate lengths respectively) and (c) comparative study of the behaviour of shells having profiles with increasing, decreasing and constant radius of curvilinear (for example catenary, cycloid and circle respectively).

Part II Consists of Chapters 5 and 6

Chapter 5 deals with classical variational methods. Here distributions, Sobolev spaces and basic results for the variational formulation of boundary value problems have been discussed. Then the variational Problem ($\mathcal{P}$) in the Galerkin form for the case of rigidly clamped open 'shallow' noncircular cylindrical shell has been formulated as follows:

$$(\mathcal{P}): \quad \text{Find } \vec{u} = (u_1, u_2, u_3) \in \vec{V} = H_0^1(\Omega) \times H_0^1(\Omega) \times H_0^2(\Omega)$$

$$\text{such that } a(\vec{u},\vec{v}) = q(\vec{v}) \quad \forall \vec{v} = (v_1, v_2, v_3) \in \vec{V}$$

where $H_0^1(\Omega)$ and $H_0^2(\Omega)$ are Sobolev spaces, $a(\cdot,\cdot): \vec{V} \times \vec{V} \to \mathbb{R}$ is the symmetric bilinear form corresponding to the energy functional and is given by : $\forall \vec{v}, \vec{w} \in \vec{V}$

$$a(\vec{v},\vec{w}) = \int_{\Omega_0} \frac{Et_0}{(1-\nu^2)} \Big[\{(1-\nu) e^{\alpha}_{\beta}(\vec{v}) e^{\beta}_{\alpha}(\vec{w}) + \nu e^{\alpha}_{\alpha}(\vec{v}) e^{\beta}_{\beta}(\vec{w})\} +$$
$$\frac{t_0^2}{12R_0^2} \{(1-\nu)\kappa^{\alpha}_{\beta}(\vec{v})\kappa^{\beta}_{\alpha}(\vec{w}) + \nu\kappa^{\alpha}_{\alpha}(\vec{v})\kappa^{\beta}_{\beta}(\vec{w})\}\Big] d\xi \, d\eta \qquad (18)$$

e^{β}_{α} and κ^{β}_{α} being the strain and curvature tensors respectively, $q(\cdot): \vec{V} \to \mathbb{R}$ is the linear form corresponding to the energy due to external forces given by

$$q(\vec{v}) = R_0^2 \int_{\Omega_0} (q_1 v_1 + q_2 v_2 + q_3 v_3)\, d\xi\, d\eta \qquad (19)$$

where $q_i \in L^2(\Omega)$, $1 \leqslant i \leqslant 3$. $\vec{V}$- ellipticity of $a(\cdot,\cdot)$ and continuity of $a(\cdot,\cdot)$ and $q(\cdot)$ are established. From the existence and uniqueness of the solution $\vec{u}$ of $(\mathcal{P})$, the equivalence of the Galerkin variational problem $(\mathcal{P})$ and the Ritz variational problem $(\mathcal{P}^*)$ given by

$$(\mathcal{P}^*): \quad \text{Find } \vec{u} \in \vec{V} \text{ such that } J(\vec{u}) = \inf_{\vec{v} \in \vec{V}} J(\vec{v})$$

$$\text{where } J(\vec{v}) = \frac{1}{2} a(\vec{v},\vec{v}) - q(\vec{v}), \qquad (20)$$

in the sense that the two problems $(\mathcal{P})$ and $(\mathcal{P}^*)$ have the same unique solution, is established. For the solution of the approximate problem $(\mathcal{P}_n^*)$ corresponding to the Ritz variational problem $(\mathcal{P}^*)$, given by

$$(\mathcal{P}_n^*): \quad \text{Find } \vec{u}_n \in \vec{V}_n \subset \vec{V} \text{ such that } J(\vec{u}_n) = \inf_{\vec{v} \in \vec{V}} J(\vec{v}_n)$$

where $J(\cdot)$ is defined by (20) and $\vec{V}_n$ is a finite dimensional subspace of the product Sobolev space $\vec{V}$, bivariate polynomials in ξ and η are chosen as the basis functions of the Rits space $\vec{V}_n$. Numerical experiments have been carried out for different choices of n and a general program has been developed for arbitrary n.

Chapter 6 deals with the conforming finite element method of solution of the variational problems $(\mathcal{P}^*)$ and $(\mathcal{P})$ using rectangular and triangular elements. For each $h > 0$ a rectangulation (respectively triangulation) $\mathcal{T}_h$ of the closed domain $\bar{\Omega} = \Omega \cup \Gamma$ into rectangles (respectively triangles) K, is associated. Corresponding to the $\mathcal{T}_h$ a finite dimensional subspace $\vec{V}_h = V_h \times V_h \times W_h$ is introduced such that

$$V_h = \{v_h : v_h \in C^{(0)}(\bar{\Omega}),\ v_h|_{K^0} \in H^1(K^0)\ \forall K \in \mathcal{T}_h$$

$$K^0 = \mathrm{int}(K), v_h|_\Gamma = 0\} \subset H^1_0(\Omega)$$

$$W_h = \{w_h : w_h \in C^{(1)}(\bar{\Omega}),\ w_h|_{K^0} \in H^2(K^0)\ \forall K \in \mathcal{T}_h$$

$$K^0 = \mathrm{int}(K),\ w_h|_\Gamma = \frac{\partial w_h}{\partial n}\Big|_\Gamma = 0\ \} \subset H^2_0(\Omega) \quad (21)$$

For the conforming finite element approximation problem ($\mathcal{P}_h^*$) in the Ritz form corresponding to $(\mathcal{P}^*)$ defined by:

($\mathcal{P}_h^*$): Find $\vec{u}_h \in \vec{V}_h$ such that $J(\vec{u}_h) = \inf\limits_{\vec{v}_h \in \vec{V}_h} J(\vec{v}_h)$

and for the conforming finite element approximation problem ($\mathcal{P}_h$) in the Galerkin form corresponding to ($\mathcal{P}$), defined by

($\mathcal{P}_h$): Find $\vec{u}_h \in \vec{V}_h$ such that $a(\vec{u}_h, \vec{v}_h) = q(\vec{v}_h)$ $\vec{v}_h \in \vec{V}_h$

the existence and uniqueness of the solution $\vec{u}_h \in \vec{V}_h \subset \vec{V}$ are easily proved when the exact integration is performed. The finite elements (K, Σ_K, P_K) are constructed for each component v_{ih} of $\vec{v}_h = (v_{1h}, v_{2h}, v_{3h})$, where Σ_K is the set of degrees of freedom for each rectangles (respectively triangle)

$K \in \mathcal{T}_h$ such that Σ_K is P_K-unisolvent, P_K being a finite dimensional linear space of real valued functions defined on $K \in \mathcal{T}_h$

For the solution of the Ritz finite element problem $(\mathcal{P}_h^*)$ using rectangular elements, for each component v_{ih} of $\vec{v}_h$ bicubic spline finite elements $(K, \Sigma_K, Q_3(K))$ have been developed, where K is a rectangle with vertices $(a_j, 1 \leqslant j \leqslant 4)$

$$\Sigma_K = (D^{(\alpha,\beta)} p(a_j),\ 1 \leqslant j \leqslant 4,\ 0 \leqslant \alpha, \beta \leqslant 1) \qquad (22)$$

is $Q_3(K)$-unisolvent, $Q_3(K)$ being the space of polynomials of degree $\leqslant 3$ in each variable ξ and η $(\dim Q_3(K) = \operatorname{card} \Sigma_K = 16)$ with a canonical basis $\{q_j^{(\alpha,\beta)},\ 1 \leqslant j \leqslant 4,\ 0 \leqslant \alpha, \beta \leqslant 1\}$ $\ni \forall g \in Q_3(K)$,

$$g = \sum_{j=1}^{4} \sum_{0 \leqslant \alpha, \beta \leqslant 1} D^{(\alpha,\beta)} g(a_j)\, q_j^{(\alpha,\beta)}. \qquad (23)$$

It has been shown that the bicubic spline $\vec{v}_{ho} \in (\mathcal{C}^{(2)}(\bar{\Omega}))^3$, $\vec{v}_{ho}|_{K^0} \in (Q_3(K^0))^3$ and $\vec{v}_{ho} \in \vec{V}_{ho}$. Numerical results are presented.

For the solution of the Galerkin finite element problem $(\mathcal{P}_h)$, Argyris', Bell's and Courant's finite elements have been developed. In the Argyris finite element (K, Σ_K, P_K) for each component v_{ih} of $\vec{v}_h$, K is a triangle with vertices $(a_j, 1 \leqslant j \leqslant 3)$ and midpoints $\{b_j, 1 \leqslant j \leqslant 3\}$

$$\Sigma_K = \{p(a_j),\ 1 \leqslant j \leqslant 3;\ Dp(a_j)(a_k - a_j),\ 1 \leqslant j, k \leqslant 3,\ j \neq k;$$

$$D^2p(a_j)(a_k-a_j,\ a_l-a_j),\quad 1 \leqslant j,\ k,\ l \leqslant 3,\quad j \neq k,\ k \neq l$$

$$\partial_n p(b_j),\ 1 \leqslant j \leqslant 3\} \qquad (24)$$

is $P_5(K)$-unisolvent, $P_5(K)$ being the space of polynomials of degree $\leqslant 5$ (dim $P_5(K)$ = card $\Sigma_K = 21$) with a basis $\{\lambda_1^l\ \lambda_2^m \times \lambda_3^n,\ l,m,n \geqslant 0,\ l+m+n = 5\}$ where $\lambda_1,\lambda_2,\lambda_3$ are barycentric coordinates.

In order to have equal number of degrees of freedom at each node of K, which is desirable from the computational point of view, Bell's finite element $(K, \Sigma'_K, P'_5(K))$ has been developed for each component v_{ih} of $\vec{v}_h$ where

$$\Sigma'_K = \{p(a_j),\ 1 \leqslant j \leqslant 3;\ Dp(a_j)(a_k-a_j),\ 1 \leqslant j,\ k \leqslant 3,\ j \neq k;$$

$$D^2p(a_j)(a_k-a_j,\ a_l-a_j),\ 1 \leqslant j,\ k, l \leqslant 3,\ j \neq k,\ k \neq l\} \qquad (25)$$

is $P'_5(K)$-inisolvent,

$P'_5(K) = \{p:\ p \in P_5(K),\ \partial_n p \in P_3[a_j,a_k]$, $[a_j,a_k]$ being the sides of K, $1 \leqslant j < k \leqslant 3\}$, (26)

dim $P'_5(K)$ = card $\Sigma'_K = 18$ and $P_4(K) \subset P'_5(K) \subset P_5(K)$.

Since for the conforming finite element method of solution it is sufficient to construct finite elements of $\mathcal{C}^{(0)}$-class for the components $v_{\alpha h}$ $(\alpha = 1,2)$ and of $\mathcal{C}^{(1)}$-class for the component v_{3h}, Courant's finite element $(K, \Sigma_K^0, P_1(K))$ for $v_{\alpha h}$ $(\alpha = 1,2)$, where $\Sigma_K = \{p(a_j),\ 1 \leqslant j \leqslant 3\}$ is $P_1(K)$- unisolvent, $P_1(K)$ being the space of polynomials of degree $\leqslant 1$ ($\dim P_1(K) = \operatorname{card} \Sigma_K = 3$) and Bell's finite element $(K, \Sigma'_K, P'_5(K))$ for v_{3h} have been developed.

Finally error estimates of the conforming finite element have been studied when the exact integration of the integrals in $a(\cdot,\cdot)$ and $q(\cdot)$ is performed.

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PART II

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