

OPTIMAL PORTFOLIO SELECTION CONTEMPLATING RISK PROPENSITY OF INVESTORS IN STOCK MARKETS

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OPTIMAL PORTFOLIO SELECTION CONTEMPLATING RISK PROPENSITY OF INVESTORS IN STOCK MARKETS

by

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To my Mother and Father

Certificate

This is to certify that the thesis entitled “**Optimal Portfolio Selection Contemplating Risk Propensity of Investors in Stock Markets**” submitted by **Ms. Amita Sharma** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bonafide research work carried out by her under my supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

It is not far fetched to see that investing in a portfolio of risky assets is wiser than investing in a single risky asset. Portfolio optimization is a well established technique in investment science to formulate a portfolio in an optimal way. Portfolio optimization models aim to pick few assets (risky) among many available to meet the investment goals which revolve primarily around return (in future) and risk (uncertainty in return). In such problems, return being a random variable is estimated by the mean value of its distribution but the daunting task is to measure the risk of uncertainty that exists in investment. Risk depends not only on the shape of the return distribution but also on the risk profile of an investor. Every investor strives hard to attain an optimal relationship between return and risk by taking the best combination of assets (optimal wight allocation) among infinite admissible portfolios.

The core issue in portfolio optimization is to describe a preference order to rank the portfolios. Broadly, there are three ranking methods in the literature: the expected utility maximization, the mean-risk optimization, and the stochastic dominance criterion. The expected utility maximization is the oldest ranking method of choice under risk. In this framework, each random variable is map to a scalar (the expected utility), and ranking is done with larger scalar value preferable. In mean-risk framework, the ranking is build on two statistics, mean value and risk value. The preference order is defined in terms of non-dominant (or efficient) solutions. The stochastic dominance (SD) ranking is inspired from the majorization theory of statistics. The first order stochastic dominance (FSD), second order stochastic dominance (SSD) and third order stochastic dominance (TSD) have practical applications in economics for their alliance respectively with rational, rational risk averse, and rational risk averse and ruin averse investors, respectively.

This thesis extends some of the available portfolio optimization models by including the risk propensity of an investor by several means. In the traditional mean-risk framework, all information from the return distribution is reflected using only two parameters and thus loosing out on many important characteristics. For example, ranking based on a single risk measure is not appropriate for investors to whom hedging against the worst outcomes (or losses) is as important as achieving the threshold target. Therefore, we propose to combine the semi-mean absolute deviation measure (to minimize the deviation around the mean return) with minimax measure (to minimize the maximum loss) in the portfolio optimization model. In the same spirit, we extend the Omega ratio (a performance index of the ratio of overdeviation from the given threshold point to the underdeviation from the same threshold point) optimization by adding an explicit risk constraint in its model with an upper bound on risk measure(s) to reflect the risk tolerance of an investor. On another instance, we replace the

constant threshold point in Omega ratio by the conditional value-at-risk at α confidence level (to control the large losses) as a stochastic target to provide a rational flavor in selecting the threshold point while minimizing the risk of large losses.

Other than mean-risk ranking, we also study the preference ranking based on SSD rule. We present two applications of SSD rule in portfolio optimization. In first one, we propose to include financial ratios in portfolio selection process and preference among portfolios are determined by SSD rule in constraint. In another instance, we propose an optimization model in the domain of enhanced indexing (a portfolio strategy to outperform the benchmark index in sense of a high mean return) with relaxed SSD criterion in the constraint. A tradeoff is created between the tolerable violation in the SSD criterion and higher mean return, thereby creating a portfolio with overall higher utility than the benchmark index.

Finally, we propose to include the market news of a stock price hitting its 52-week high point along with its company's past performance (in terms of profit) in portfolio selection strategy. Such momentum strategies are active portfolio strategies designed to get benefit out of the weak efficient form of the underlying markets. The proposed strategy is found to perform significantly better in the bearish scenarios and thus succeeded in hedging the risk of large losses.

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List of Abbreviations

AER	Average excess return
ASD	Almost stochastic dominance
ASSD	Almost second stochastic dominance
$CVaR_\alpha$	Conditional Value-at-Risk at α degree of confidence level
EI	Enhanced indexing
FA	Financial analysis
FRs	Fundamental ratios
GAMS	General algebraic modeling system
IT	Index tracking
LP	Linear program
MAD	Mean absolute deviation
MR	Mean-risk
MM	Minimax
MSR	Modified Sharpe ratio
RO	Robust optimization
SemiMAD	Semi-mean absolute deviation
SD	Stochastic dominance
SSD	Second order stochastic dominance
SOCP	Second order cone program
SR	Sharpe ratio
SPO	Sectoral portfolio optimization
VaR_α	Value-at-Risk at α degree of confidence level