

**DYNAMIC BEHAVIOUR OF INSTRUMENTED
SINGLE VERTICAL AND BATTER PILES AND
PILE GROUPS UNDER VERTICAL AND COUPLED
VIBRATIONS: FIELD MODEL TESTING AND
THEORETICAL STUDY**

ROHIT RALLI



**DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI
SEPTEMBER 2021**

**DYNAMIC BEHAVIOUR OF INSTRUMENTED SINGLE
VERTICAL AND BATTER PILES AND PILE GROUPS
UNDER VERTICAL AND COUPLED VIBRATIONS: FIELD
MODEL TESTING AND THEORETICAL STUDY**

by

ROHIT RALLI

Department of Civil Engineering

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

SEPTEMBER 2021

CERTIFICATE

This is to certify that the thesis entitled “**Dynamic Behaviour of Instrumented Single Vertical and Batter Piles and Pile Groups under Vertical and Coupled Vibrations: Field Model Testing and Theoretical Study**”, submitted by **Mr. Rohit Ralli** to the Indian Institute of Technology Delhi, is a record of bonafide research work carried out by him under my supervision and is worthy of consideration for the award of **Doctor of Philosophy** degree. The research reported in this thesis has not been submitted in part or full to any other university or institute for the award of any degree or diploma.

Prof. Bappaditya Manna

Professor

Department of Civil Engineering

Indian Institute of Technology Delhi

New Delhi- 110016, INDIA

Prof. Manoj Datta

Emeritus Professor

Department of Civil Engineering

Indian Institute of Technology Delhi

New Delhi- 110016, INDIA

ACKNOWLEDGMENTS

This research work could not have been possible without the support of a number of individuals and I would like to extend my sincere thanks to them. First and foremost, I would like to express my immense gratitude towards my respected supervisors, Prof. Bappaditya Manna and Prof. Manoj Datta for their invaluable guidance, support, inspiration, encouragement, motivation, affection and for providing all assistance required to complete this work. Their painstaking efforts and meticulous supervision in each and every phase of this research work for its improvement are gratefully acknowledged. I express my sincere thanks and gratitude to Student Research Committee members Prof. B.R Chahar, Prof. J.T. Shahu and Prof. Maloy K. Singha for their valuable suggestions at various stages of my research work.

I express my gratitude to Prof. G.V. Ramana, Head of Department of Civil Engineering, who encouraged me in many ways and provided all necessary facilities to help in carrying out the present investigation. I thankfully acknowledge the encouragement and support given by faculty members of the Department of Civil Engineering especially, Prof. R. Ayothiraman, Prof. T. Chakraborty, Prof. K.S. Rao and Dr. Prashanth Vangala throughout the tenure of my research. I am really grateful to the laboratory staff members of the

Geotechnical and Geoenvironmental section for their help to bring this project into reality. I am thankful to Mr. Manoj Kumar Nilam and Mr. Alok Kumar for their valuable assistance at various stages of the experimental work. I would also like to thank Mr. Anand, Mr. Tikaram, Mr. Gopi and Mr. Ram Janam for their help during the field tests.

I must thank my co-research scholar friends Dr. Sanjit Biswa, Dr. Shiva Shankar Choudhary, Dr. Ankesh Kumar, Dr. Aditya Singh, Ms. Kavita Tandon, Mr. Prasun Halder, Ms. Sujata Fulambakar for their selfless help, cooperation and continuous moral support during my stay at IIT Delhi. I would like to express my gratitude to all my friends and well-wishers who have directly or indirectly contributed in the course of my research work.

I am greatly indebted to my family for their continual love, support and blessings to see me scaling greater heights of life. I am very much grateful to my parents Mr. Praveen Ralli and Mrs. Sadhna Ralli, my wife and daughter Reetika Ralli and Radhya Ralli, my elder brother and his wife Sunny Ralli and Shanu Ralli and little ones Hritwik and Miraya Ralli, my sister and his husband Tanya and Indranil Joshi for all their sacrifices, incredible patience and understanding during my entire research period. I would also like to thank my parents-in-law for their continuous encouragement, blessing and

unconditional support. Their continuous encouragement, love, blessings, endless moral and emotional supports provide me strength throughout the journey without which I could not have accomplished this task.

Finally, I shall remember forever my Late grandmother, with her blessing and support I was not able to accomplished my task.

Place: IIT Delhi

Date:

(Rohit Ralli)

ABSTRACT

In the present study, the response of a single vertical pile, $P1$ ($\beta = 0^\circ$) and single batter piles, $P2$ ($\beta = 10^\circ$) and $P3$ ($\beta = 20^\circ$) and 2 x 2 vertical pile group (VPG) and batter pile group (BPG) for different dynamic force level under vertical and coupled vibrations are investigated. All the piles in BPG are unilaterally inclined at an angle of 20° with vertical. The centre to centre spacing between the piles are kept at a distance of $3d$ for both the pile groups. The dynamic test results are presented in terms of the frequency versus amplitude response, frequency versus contact pressure response, distribution of dynamic bending moments and axial loads along the pile in vertical and batter piles and pile groups. Comparative assessment of test results of both the vertical and batter piles with respect to the peak resonant frequency and resonant amplitudes, pile soil separation length, dynamic axial loads and bending moments under vertical and coupled vibrations has been emphasized. The effect of loading direction i.e., coupled vibration in along and across the inclination of batter pile are also compared in a comprehensive manner for single batter piles and pile groups.

To investigate the complex nonlinear characteristics of the soil-pile system for single vertical and batter piles and pile groups under machined induced vertical and coupled (horizontal and rocking) vibrations, the field dynamic test results are compared with the theoretical results. Two soil models are used to study the linear and nonlinear behaviour of the vertical and batter piles under dynamic loading. Nonlinear responses of the piles are predicted by introducing the weak boundary zone parameters and soil-pile separation length for different excitation forces. The variation of frequency dependent stiffness and damping of single vertical and batter piles and pile groups are also investigated and

compared for different excitation intensities under vertical and coupled vibrations using the theoretical approaches. The pile-soil-pile interaction phenomenon of the 2×2 vertical and batter pile group are also investigated by calculating the group efficiency ratios (GER) of the pile group for different excitation intensities and vibration modes.

Various design charts are proposed based on the finding of the study, to estimate the soil-pile separation lengths and boundary zone parameters for the single vertical and batter piles and 2×2 pile group with different pile batter angles under vertical and coupled vibrations. The applicability of the proposed design charts is validated for full-scale vertical and batter prototype piles using scaling law. It is found that the proposed design curves can be used to determine the nonlinear frequency-amplitude responses of the full-scale pile foundations under both vertical and coupled vibrations for similar soil-pile conditions. The charts are also validated using experimental data of various literature to check the reliability of the design charts. It is found that the proposed design charts can be used to determine the nonlinear frequency versus amplitude responses of the pile foundations under vertical and coupled vibrations for similar type of soil-pile conditions.

Keywords: Vertical pile, Batter pile, vertical vibration, coupled vibration, soil-pile separation, dynamic axial load, dynamic bending moment, linear approach, nonlinear approach, stiffness, damping, loading direction, GER.

सार

वर्तमान अध्ययन में, सिंगल वर्टिकल पाइल, $P1$ ($\theta = 0^\circ$) और सिंगल बैटर पाइल्स, $P2$ ($\theta = 10^\circ$) और $P3$ ($\theta = 20^\circ$) और 2×2 वर्टिकल पाइल ग्रुप (VPG) और बैटर पाइल की प्रतिक्रिया ऊर्ध्वाधर और युग्मित कंपनों के तहत विभिन्न गतिशील बल स्तर के लिए समूह (बीपीजी) की जांच की जाती है। बीपीजी में सभी ढेर ऊर्ध्वाधर के साथ 20° के कोण पर एकतरफा झुके हुए हैं। ढेर के बीच केंद्र से केंद्र की दूरी दोनों ढेर समूहों के लिए 3 डी की दूरी पर रखी जाती है। गतिशील परीक्षण के परिणाम आवृत्ति बनाम आयाम प्रतिक्रिया, आवृत्ति बनाम संपर्क दबाव प्रतिक्रिया, गतिशील झुकने वाले क्षणों के वितरण और ऊर्ध्वाधर और बैटर ढेर और ढेर समूहों में ढेर के साथ अक्षीय भार के संदर्भ में प्रस्तुत किए जाते हैं। शिखर गुंजयमान आवृत्ति और गुंजयमान आयामों, ढेर मिट्टी की जुदाई लंबाई, गतिशील अक्षीय भार और ऊर्ध्वाधर और युग्मित कंपन के तहत झुकने वाले क्षणों के संबंध में ऊर्ध्वाधर और बल्लेबाज पाइल दोनों के परीक्षण परिणामों के तुलनात्मक मूल्यांकन पर जोर दिया गया है। लोडिंग दिशा के प्रभाव यानी, बैटर पाइल के झुकाव के साथ-साथ युग्मित कंपन की तुलना सिंगल बैटर पाइल्स और पाइल समूहों के लिए व्यापक तरीके से की जाती है।

मशीनी प्रेरित ऊर्ध्वाधर और युग्मित (क्षैतिज और रॉकिंग) कंपन के तहत एकल ऊर्ध्वाधर और बल्लेबाज ढेर और ढेर समूहों के लिए मिट्टी-ढेर प्रणाली की जटिल गैर-रेखीय विशेषताओं की जांच करने के लिए, सैद्धांतिक परिणामों के साथ क्षेत्र गतिशील परीक्षण परिणामों की तुलना की जाती है। डायनेमिक लोडिंग के तहत वर्टिकल और बैटर पाइल्स के लीनियर और नॉनलाइनियर व्यवहार का अध्ययन करने के लिए दो मृदा मॉडल का उपयोग किया जाता है। विभिन्न उत्तेजना बलों के लिए कमजोर सीमा क्षेत्र मापदंडों और मिट्टी-ढेर पृथक्करण लंबाई को शुरू करके बवासीर की गैर-

रेखीय प्रतिक्रियाओं की भविष्यवाणी की जाती है। एकल ऊर्ध्वाधर और बल्लेबाज ढेर और ढेर समूहों की आवृत्ति निर्भर कठोरता और नमी की भिन्नता की भी जांच की जाती है और सैद्धांतिक दृष्टिकोण का उपयोग करके लंबवत और युग्मित कंपन के तहत विभिन्न उत्तेजना तीव्रता के लिए तुलना की जाती है। विभिन्न उत्तेजना तीव्रता और कंपन मोड के लिए ढेर समूह के समूह दक्षता अनुपात (जीईआर) की गणना करके 2×2 लंबवत और बल्लेबाज ढेर समूह की ढेर-मिट्टी-ढेर बातचीत घटना की भी जांच की जाती है।

सिंगल वर्टिकल और बैटर पाइल्स के लिए मिट्टी-पाइल पृथक्करण लंबाई और सीमा क्षेत्र मापदंडों का अनुमान लगाने के लिए अध्ययन की खोज के आधार पर विभिन्न डिज़ाइन चार्ट प्रस्तावित किए गए हैं और ऊर्ध्वाधर और युग्मित कंपन के तहत विभिन्न पाइल बैटर कोणों के साथ 2×2 पाइल समूह हैं। प्रस्तावित डिज़ाइन चार्ट की प्रयोज्यता स्केलिंग कानून का उपयोग करके पूर्ण पैमाने पर लंबवत और बैटर प्रोटोटाइप ढेर के लिए मान्य है। यह पाया गया है कि प्रस्तावित डिज़ाइन घटता का उपयोग समान मिट्टी-ढेर स्थितियों के लिए ऊर्ध्वाधर और युग्मित कंपन दोनों के तहत पूर्ण पैमाने पर ढेर नींव की गैर-आवृत्ति आवृत्ति-आयाम प्रतिक्रियाओं को निर्धारित करने के लिए किया जा सकता है। डिज़ाइन चार्ट की विश्वसनीयता की जांच करने के लिए विभिन्न साहित्य के प्रयोगात्मक डेटा का उपयोग करके चार्ट को भी मान्य किया जाता है। यह पाया गया है कि प्रस्तावित डिज़ाइन चार्ट का उपयोग समान प्रकार की मिट्टी-ढेर स्थितियों के लिए ऊर्ध्वाधर और युग्मित कंपनों के तहत ढेर नींव की गैर-रेखीय आवृत्ति बनाम आयाम प्रतिक्रियाओं को निर्धारित करने के लिए किया जा सकता है।

कीवर्ड: वर्टिकल पाइल, बैटर पाइल, वर्टिकल वाइब्रेशन, कपल्ड वाइब्रेशन, सॉइल-पाइल सेपरेशन, डायनेमिक एक्सियल लोड, डायनेमिक बेंडिंग मोमेंट, लीनियर अप्रोच, नॉनलाइनियर अप्रोच, स्टिफनेस, डंपिंग, लोडिंग डायरेक्शन, जीईआर।

CONTENTS

Certificate	i
Acknowledgements	iii
Abstract	vii
संक्षेप	ix
List of Content	xiii
List of Figures	xxi
List of Tables	xxxix
List of Symbols	xlv
CHAPTER 1 INTRODUCTION	1
1.1 General	1
1.2 Objectives of the study	8
1.3 Scope of the study	9
1.4 Research methodology	10
1.5 Organization of the thesis	10
CHAPTER 2 REVIEW OF LITERATURE	13
2.1 General	13
2.2 Dynamic Response of Vertical Pile Foundations: Theoretical Study	13
2.2.1 Equivalent cantilever method	14
2.2.2 Mass-spring-dashpot model	15
2.2.3 Winkler foundation model	16
2.2.4 Cone model	19
2.2.5 Elastic Continuum approach	20
2.2.6 Finite element method	26
2.2.7 Boundary element method	30
2.2.8 Summary	31
2.3 Dynamic Response of Vertical Pile Foundations: Experimental Study	31
2.3.1 Full scale field test	32

2.3.2	Small prototype field test	35
2.3.3	Small scale laboratory test	37
2.3.4	Summary	39
2.4	Dynamic Response of Batter Pile Foundations: Theoretical study	40
2.4.1	Boundary element method	40
2.4.2	Finite element method	41
2.4.3	Boundary element method-finite element method	44
2.5	Dynamic Response of Batter Pile Foundations: Experimental study	47
2.5.1	Summary of experimental and theoretical studies	54
CHAPTER 3	EXPERIMENTAL INVESTIGATION	55
3.1	General	55
3.2	Site Investigation	56
3.2.1	Standard Penetration test (SPT)	56
3.2.2	Multichannel analysis of surface waves (MASW)	56
3.3.3	Laboratory testing	58
3.3	Experimental setup	62
3.3.1	Test piles	62
3.3.2	Instrumentation of test piles	64
3.3.2.1	Pressure sensor	64
3.3.2.2	Strain gauge	66
3.3.3	Installation of test piles	68
3.3.3.1	Single piles	68
3.3.3.2	Pile groups	69
3.3.3.3	Test body	73
3.3.4	Description of dynamic testing instruments	73
3.3.4.1	Mechanical oscillator	73
3.3.4.2	Ac motor with frequency indicating sensor	74
3.3.4.3	Flexible shaft	75
3.3.4.4	Speed control unit	76
3.3.4.5	Vibration measurement	77
3.3.5	Testing program	78
3.4	Forced vibration testing setup	80
3.4.1	Single vertical and batter piles	80
3.4.1.1	Vertical vibration testing setup	81
3.4.1.2	Coupled vibration testing setup	85
3.4.2	2 × 2 Vertical and batter pile groups	88
3.4.2.1	Vertical vibration testing setup	88
3.4.2.2	Coupled vibration testing setup	90
3.5	Forced Vibration Testing Results	93

3.5.1	Single vertical and batter piles	93
3.5.1.1	Vertical vibration test results	94
3.5.1.1.1	Frequency-amplitude response	94
3.5.1.1.2	Soil-pile separation	104
3.5.1.1.3	Dynamic axial load	114
3.5.1.1.4	Dynamic bending moment	116
3.5.1.2	Coupled vibration test results	118
3.5.1.2.1	Frequency-amplitude response	118
3.5.1.2.2	Soil-pile separation	132
3.5.1.2.3	Dynamic bending moment	136
3.5.1.2.4	Dynamic axial load	140
3.5.2	2 × 2 Vertical and batter pile groups	141
3.5.2.1	Vertical vibration test results	142
3.5.2.1.1	Frequency-amplitude response	142
3.5.2.1.2	Dynamic axial load	148
3.5.2.1.3	Dynamic bending moment	150
3.5.2.1.4	Soil-pile separation	151
3.5.2.2	Coupled vibration test results	154
3.5.2.2.1	Frequency-amplitude response	154
3.5.2.2.2	Soil-pile separation	165
3.5.2.2.3	Dynamic bending moment	170
3.6	Summary	172
CHAPTER 4	THEORETICAL STUDY	175
4.1	General	175
4.2	Linear approach	176
4.2.1	Impedance function of single vertical pile	176
4.2.1.1	Element stiffness matrix	177
4.2.1.1.1	Vertical vibration	178
4.2.1.1.2	Horizontal and rocking vibrations	179
4.2.1.1.3	Pile tip reaction	181
4.2.1.2	Complex stiffness of pile	181
4.2.1.2.1	Vertical stiffness of pile	182
4.2.1.2.2	Horizontal stiffness of pile	183
4.2.2	Impedance function of single batter pile	185
4.3	Nonlinear approach	187

4.3.1	Impedance function of single vertical pile	187
4.3.1.1	Stiffness of composite medium	189
4.3.1.1.1	Vertical stiffness	190
4.3.1.1.2	Horizontal stiffness	191
4.3.1.1.3	Rocking stiffness	193
4.3.1.2	Stiffness of pile in composite medium	193
4.3.2	Impedance function of single batter pile	195
4.4	Analysis of pile groups	196
4.4.1	Impedance function of vertical pile group	196
4.4.1.1	Static interaction factor	199
4.4.1.2	Dynamic interaction factor	200
4.4.2	Impedance function of battered pile group	200
4.5	Frequency-amplitude response of pile foundation	203
4.5.1	Vertical response of pile foundation	204
4.5.2	Coupled response of pile foundation	205
4.6	Stiffness and damping of pile	206
4.6.1	Stiffness and damping of pile in vertical vibration	207
4.6.1.1	Single vertical pile and pile group	207
4.6.1.2	Single batter piles and pile group	210
4.6.1.3	Comparison of stiffness and damping for different batter angles	214
4.6.2	Stiffness and damping of pile in coupled vibration	217
4.6.2.1	Single vertical pile and pile group	217
4.6.2.2	Single batter piles and pile group	222
4.6.2.3	Comparison of stiffness and damping for different batter angles	230
4.7	Group efficiency ratio	238
4.8	Summary	240

CHAPTER 5	THEORY VERSUS EXPERIMENT	243
5.1	General	243
5.2	Theory versus experiment: Vertical vibration	244
5.2.1	Comparison with linear approach	244
5.2.1.1	Single vertical and vertical pile group	244
5.2.1.2	Single batter piles and batter pile group	247
5.2.2	Comparison with nonlinear approach	249
5.2.2.1	Single vertical and vertical pile group	249
5.2.2.2	Single batter piles and batter pile group	258
5.2.2.2.1	Nonlinear approach 1	259

	5.2.2.2.2	Nonlinear approach 2	261
	5.2.2.3	Comparison of boundary zone parameters and dynamic response between vertical pile and batter pile	266
5.3		Theory versus experiment: Coupled vibration	273
	5.3.1	Comparison with linear approach	273
	5.3.1.1	Single vertical and vertical pile group	273
	5.3.1.2	Single batter piles and batter pile group	274
	5.3.2	Comparison with nonlinear approach	277
	5.3.2.1	Single vertical and vertical pile group	277
	5.3.2.2	Single batter piles and batter pile group	281
	5.3.2.2.1	Nonlinear approach 1	281
	5.3.2.2.2	Nonlinear approach 2	283
	5.3.2.3	Comparison of boundary zone parameters and dynamic response between vertical pile and batter pile	288
5.4		Theory versus experiment: Single batter piles and batter pile group under coupled vibration across the pile batter angle	292
	5.4.1	Comparison with linear approach	293
	5.4.2	Comparison with nonlinear approach 1	294
	5.4.3	Effect of loading direction on dynamic response of single batter piles and batter pile group	297
5.5		Load transfer mechanism	305
	5.5.1	Dynamic axial load along the pile	305
	5.5.1.1	Single vertical pile and vertical pile group	305
	5.5.1.2	Single batter piles and batter pile group	306
	5.5.2	Dynamic bending moment along the pile	307
5.6		Separation length of pile and soil	310
5.7		Summary	318
CHAPTER 6 DESIGN CHARTS			321
6.1		General	321
6.2		Design charts	322
	6.2.1	Weak boundary zone parameters	322
6.3		Validation of design charts using scaling laws	342
	6.3.1	Comparison of prototype and mode pile responses	345
6.4		Validation of design charts using literature	358
6.5		Design problem	360

6.6	Summary	366
CHAPTER 7	CONCLUSIONS	369
7.1	Summary	369
7.2	Conclusions from experimental results	369
7.2.1	Frequency-amplitude response of single vertical pile compared with single batter piles with different pile batter angles of 10° and 20°	370
7.2.1.1	Vertical vibrations	370
7.2.1.2	Coupled vibrations	371
7.2.2	Frequency-amplitude response of 2 × 2 vertical pile group and 2 × 2 batter pile group with pile batter angle of 20 degrees	373
7.2.2.1	Vertical vibrations	373
7.2.2.2	Coupled vibrations	374
7.2.3	Soil-pile separation lengths of single vertical and batter piles as well as vertical and batter pile groups with different batter angles	375
7.2.3.1	Vertical vibrations: Single pile	375
7.2.3.2	Coupled vibrations: Single pile	375
7.2.3.3	Vertical vibrations: pile groups	376
7.2.3.4	Coupled vibrations: pile groups	376
7.2.4	Distribution of dynamic axial loads and dynamic bending moments along the single piles and pile groups with different batter angles	376
7.2.4.1	Vertical vibrations: Single pile	376
7.2.4.2	Coupled vibrations: Single pile	377
7.2.4.3	Vertical vibrations: pile groups	377
7.2.4.4	Coupled vibrations: pile groups	378
7.3	Conclusion from theoretical study	378
7.3.1	Frequency dependent stiffness and damping parameters of the soil-pile system and group efficiency ratio (GER) of the pile group	379
7.4	Conclusions from theory versus experiment	382
7.4.1	Prediction of nonlinear response of soil-pile system by evaluating the weak boundary soil zone parameters and soil-pile separation length	383
7.5	Conclusions from Design charts	387
7.6	Take away message from Research work	388
7.7	Research work enhancing literature	389
7.8	Main/stand-alone contribution	390

7.9	Significant contributions of the Thesis	390
7.7	Recommendations for the Future research	392
REFERENCES		393
Curriculum Vitae		421

LIST OF FIGURES

Fig. No.	Description	Page No.
1.1	Batter piles (a) positive (out) batter pile; and (b) negative (in) batter pile	2
1.2	Flow chart of the present research methodology	12
3.1	Plan view of test piles, MASW tests, and borehole	59
3.2	Multichannel analysis channel (MASW) testing setup	60
3.3	Grain size distribution of soil for different depths of the soil layers	61
3.4	Soil profile with SPT-N values, field SPT <i>N</i> values and shear wave velocity profile	61
3.5	Test piles	64
3.6	Pressure sensors with Data acquisition system	66
3.7	Instrumentation (a) pressure sensors; (b) strain gauges in both longitudinal and lateral direction; (c) layout of wires in test pile	68
3.8	Step wise installation of a vertical pile	70
3.9	Installation of batter piles with pile inclination of 20 degrees (a) guide rod; (b) installation of guide rod; (c) pile driven at required pile batter angle with the help of SPT hammer; (d) installed batter pile with pile cap; (e) installed batter pile loading assembly.	71
3.10	Installation of batter pile group (BPG) (a) installed batter pile group and circular guide sleeve; (b) installed batter pile group with pile cap connectors; (c) batter pile group with loading assembly.	72
3.11	(a) Mechanical oscillator (b) eccentric masses of oscillator	74
3.12	AC motor with attached frequency sensor	75
3.13	Flexible shaft with steel coupling on both ends	76
3.14	Speed control unit	77
3.15	Uniaxial accelerometer	78
3.16	Schematic diagram of pile cap, steel plates and oscillator assembly for vertical vibration of pile as shown in Table 3.3.	82

3.17	Schematic diagram of vertical vibration testing setup of batter pile ($\beta = 20^\circ$) with instrumentation	84
3.18	Complete field vertical vibration test set up of batter piles	85
3.19	Plan view of direction of coupled vibration (a) along (x direction); and (b) across (y direction) the pile batter angle	86
3.20	Schematic diagram coupled vibration testing setup of batter pile ($\beta = 20^\circ$) with instrumentation	87
3.21	Complete vibration field testing set up of batter pile	88
3.22	Pile groups geometry in plane for (a) 2 x 2 VPG; and (b) 2 x 2 BPG for vertical vibration	89
3.23	Schematic diagram of vertical vibration testing setup of batter pile group with instrumentation	90
3.24	Plan and elevation view of pile group geometry (VPG) for (a) 2 x 2 vertical pile group; and (b) 2 x 2 batter pile group (BPG) for coupled vibration	91
3.25	Schematic diagram of coupled vibration testing setup of batter pile group (BPG) with instrumentation	92
3.26	Complete coupled vibration field testing set up of batter pile group (BPG)	93
3.27	Typical measured time vs acceleration response of single vertical pile, <i>P1</i> ($\beta = 0^\circ$) for excitation intensity (<i>W.e</i>) of 1.454 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	95
3.28	Typical measured time vs acceleration response of single batter pile, <i>P2</i> ($\beta = 10^\circ$) for excitation intensity (<i>W.e</i>) of 1.454 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	96
3.29	Typical measured time vs acceleration response of single batter pile, <i>P3</i> ($\beta = 20^\circ$) for excitation intensity (<i>W.e</i>) of 1.454 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz.	97
3.30	Experimental vertical response curves under 12 kN: (a) Vertical pile, <i>P1</i> ($\beta=0^\circ$); (b) Batter pile, <i>P2</i> ($\beta= 10^\circ$); (c) Batter pile, <i>P3</i> ($\beta=20^\circ$).	98
3.31	Experimental dimensionless vertical response curves under 12 kN: (a) Vertical pile, <i>P1</i> ($\beta=0^\circ$); (b) Batter pile, <i>P2</i> ($\beta= 10^\circ$); (c) Batter pile, <i>P3</i> ($\beta=20^\circ$).	100

3.32	Variation of (a) Dimensionless Resonant frequency and (b) Dimensionless Resonant amplitude with excitation intensity ($W.e$) for different pile batter angle (β) with static load (W_s) of 12 kN.	102
3.33	Variation of (a) Dimensionless resonant frequency and (b) Dimensionless resonant amplitude with different pile batter angle (β) for different excitation intensities ($W.e$) for static load (W_s) of 12 kN.	104
3.34	Typical measured time vs contact pressure of single vertical pile, $P1$ ($\beta = 0^\circ$) for excitation intensity ($W.e$) of 1.454 N-m at 0.3-m depth (a) 20 Hz (b) 25 Hz (c) 30 Hz (d) 35 Hz	107
3.35	Typical measured time vs contact pressure of single batter pile, $P2$ ($\beta = 10^\circ$) for excitation intensity ($W.e$) of 1.454 N-m at 0.3-m depth (a) 20 Hz (b) 25 Hz (c) 30 Hz (d) 35 Hz.	108
3.36	Typical measured time vs contact pressure of single batter pile, $P3$ ($\beta = 20^\circ$) for excitation intensity ($W.e$) of 1.454 N-m at 0.3-m depth (a) 20 Hz (b) 25 Hz (c) 30 Hz (d) 35 Hz	109
3.37	Contact pressure versus frequency for vertical pile, $P1$ ($\beta = 0^\circ$) (a) $W.e = 0.225$ N-m; (b) $W.e = 0.885$ N-m; (c) $W.e = 1.454$ N-m; (d) $W.e = 1.944$ N-m under static load (W_s) of 12 kN	111
3.38	Contact pressure versus frequency for batter pile, $P2$ ($\beta = 10^\circ$) (a) $W.e = 0.225$ N-m; (b) $W.e = 0.885$ N-m; (c) $W.e = 1.454$ N-m; (d) $W.e = 1.944$ N-m under static load (W_s) of 12 kN	112
3.39	Contact pressure versus frequency for batter pile, $P3$ ($\beta = 20^\circ$) (a) $W.e = 0.225$ N-m; (b) $W.e = 0.885$ N-m; (c) $W.e = 1.454$ N-m; (d) $W.e = 1.944$ N-m under static load (W_s) of 12 kN	113
3.40	Experimental load distribution in a pile; (a) Vertical pile, $P1$ ($\beta=0^\circ$); (b) Batter pile, $P2$ ($\beta=10^\circ$); (c) Batter pile, $P3$ ($\beta=20^\circ$) under a static load (W_s) of 12 kN	116
3.41	Experimental bending moment in pile; (a) Vertical pile, $P1$ ($\beta = 0^\circ$); (b) Batter pile, $P2$ ($\beta = 10^\circ$); (c) Batter pile, $P3$ ($\beta = 20^\circ$) under a static load (W_s) of 12 kN.	117

3.42	Typical measured time vs horizontal acceleration response (x direction) of single batter pile, $P2$ ($\beta = 10^\circ$) for excitation intensity ($W.e$) of 1.454 N-m (a) 20 Hz; (b) 25 Hz; (c) 30 Hz; (d) 35 Hz	120
3.43	Typical measured time vs rocking acceleration response (x direction) of single batter pile, $P2$ ($\beta = 10^\circ$) for excitation intensity ($W.e$) of 1.454 N-m (a) 20 Hz; (b) 25 Hz; (c) 30 Hz; (d) 35 Hz	121
3.44	Experimental frequency versus amplitude response curves of vertical pile, $P1$ ($\beta = 0^\circ$) subjected to coupled vibration along (x direction) the pile batter angle under 12 kN: (a) horizontal component (b) rocking component	122
3.45	Experimental frequency versus amplitude response curves of batter pile, $P2$ ($\beta = 10^\circ$) subjected to coupled vibration along (x direction) the pile batter angle under 12 kN: (a) horizontal component (b) rocking component	122
3.46	Experimental frequency versus amplitude response curves of batter pile, $P3$ ($\beta = 20^\circ$) subjected to coupled vibration along (x direction) the pile batter angle under 12 kN: (a) horizontal component (b) rocking component.	123
3.47	Variation of (a) First peak resonant frequency (b) Second peak resonant frequency (c) Maximum horizontal resonant amplitude (d) Maximum rocking resonant amplitude with excitation intensity ($W.e$) for different pile batter angle subjected to coupled vibration along (x direction) the pile batter angle for static load (W_s) of 12 kN.	126
3.48	Variation of (a) First peak resonant frequency (b) Second peak resonant frequency (c) Maximum horizontal resonant amplitude (d) Maximum rocking resonant amplitude with different pile batter angle subjected to coupled vibration along (x direction) the pile batter angle for different excitation intensities ($W.e$) for static load (W_s) of 12 kN.	128
3.49	Variation of (a) First peak resonant frequency (b) Second peak resonant frequency (c) Maximum horizontal resonant amplitude (d) Maximum rocking resonant amplitude with excitation intensity ($W.e$) for batter piles, $P2$ and $P3$ subjected to coupled vibration along (x direction) and across (y direction) the pile batter angle with static load (W_s) of 12 kN.	130

3.50	Variation of (a) First peak resonant frequency (b) Second peak resonant frequency (c) Maximum horizontal resonant amplitude (d) Maximum rocking resonant amplitude with static load (W_s) for different pile batter angle subjected to coupled vibration along (x direction) the pile batter angle for different excitation intensities ($W.e$) of 0.225 N-m and 1.944 N-m.	132
3.51	Contact pressure versus frequency for batter pile, $P2$ ($\beta = 10^\circ$) subjected to coupled vibration (a) $W.e = 0.225$ N-m; (b) $W.e = 0.885$ N-m; (c) $W.e = 1.454$ N-m; and (d) $W.e = 1.944$ N-m along (x direction) the pile batter angle with static load (W_s) of 12 kN.	136
3.52	Distribution of measured dynamic bending strain against normalised frequency along single batter pile, $P3$ ($\beta = 20^\circ$) subjected to coupled vibration along (x direction) the pile batter angle at excitation intensity ($W.e$) of 1.454 N-m.	137
3.53	Measured dynamic bending moment along (a) Vertical pile, $P1$, ($\beta = 0^\circ$) (b) Batter pile, $P2$, ($\beta = 10^\circ$) (c) Batter pile, $P3$, ($\beta = 20^\circ$) subjected to coupled vibration along (x direction) the batter pile under a static load (W_s) of 12 kN.	138
3.54	Variation of maximum dynamic bending moment with different pile batter angle subjected to coupled vibration along (x direction) the pile batter angle with static load (W_s) of 12 kN	139
3.55	Variation of maximum dynamic bending moment for batter piles, $P2$ and $P3$ subjected to coupled vibration along (x direction) and across (y direction) for different excitation intensities ($W.e$) with static load (W_s) of 12 kN	140
3.56	Measured dynamic axial load along (a) Vertical pile, $P1$, ($\beta = 0^\circ$) (b) Batter pile, $P2$, ($\beta = 10^\circ$) (c) Batter pile, $P3$, ($\beta = 20^\circ$) subjected to coupled vibration along (x direction) the pile batter angle under a static load (W_s) of 12 kN.	141
3.57	Typical measured time-acceleration response of VPG for excitation intensity ($W.e$) of 1.944 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	144

3.58	Typical measured time versus acceleration response of BPG for excitation intensity ($W.e$) of 1.944 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	145
3.59	Experimental frequency versus amplitude response curves subjected to vertical vibration under 12 kN: (a) VPG; and (b) BPG under a static load (W_s) of 12 kN	146
3.60	Variation of (a) resonant frequency (f_{gz}); (b) resonant amplitude (A_{gz}) with excitation intensity ($W.e$) for VPG and BPG for static load (W_s) of 12 kN.	148
3.61	Measured dynamic axial load along the piles for (a) VPG and (b) BPG for different excitation intensity ($W.e$) under a static load (W_s) of 12 kN.	149
3.62	Measured dynamic bending moment along the piles for BPG for different excitation intensity ($W.e$) under a static load (W_s) of 12 kN.	151
3.63	Typical measured time versus contact pressure of BPG for excitation intensity ($W.e$) of 1.944 N-m at a depth of 0.3 m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	153
3.64	Maximum contact pressure versus frequency for (a) BPG; (b) VPG for excitation intensity ($W.e$) of 1.944 N-m under a static load (W_s) of 12 kN	154
3.65	Typical measured time vs horizontal acceleration response of BPG subjected to coupled vibration along x direction for excitation intensity ($W.e$) of 1.944 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	156
3.66	Typical measured time vs rocking acceleration response of BPG subjected to coupled vibration along x direction for excitation intensity ($W.e$) of 1.944 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	157
3.67	Experimental frequency versus amplitude response curves of VPG subjected to coupled vibration along x direction for different excitation intensities ($W.e$) under a static load of 12 kN: (a) horizontal component (b) rocking component	158
3.68	Experimental frequency versus amplitude response curves of BPG subjected to coupled vibration along x direction for different excitation intensities ($W.e$) under a static load of 12 kN: (a) horizontal component (b) rocking component	158

3.69	Experimental frequency versus amplitude response curves of BPG subjected to coupled vibration along y direction under a static load of 12 kN: (a) horizontal component (b) rocking component	160
3.70	Comparison of (a) First peak resonant frequency; and (b) Maximum horizontal resonant amplitude for VPG and BPG subjected to horizontal excitation in x direction for different excitation intensities ($W.e$) for static load (W_s) of 12 kN.	163
3.71	Comparison of (a) First peak resonant frequency; and (b) Maximum horizontal resonant amplitude for BPG-x and BPG-y subjected to coupled vibration for different excitation intensities ($W.e$) for static load (W_s) of 12 kN.	165
3.72	Typical measured time vs contact pressure response of BPG subjected to coupled vibration in x direction for excitation intensity ($W.e$) of 1.944 N-m (a) 25 Hz; (b) 30 Hz; (c) 35 Hz; (d) 40 Hz	166
3.73	Contact pressure versus frequency for VPG subjected to coupled vibration in x direction for excitation intensities (a) $W.e = 0.225$ N-m; (b) $W.e = 0.885$ N-m; (c) $W.e = 1.454$ N-m; and (d) $W.e = 1.944$ N-m with static load (W_s) of 12 kN.	168
3.74	Contact pressure versus frequency for BPG subjected to coupled vibration in x direction for excitation intensities (a) $W.e = 0.225$ N-m; (b) $W.e = 0.885$ N-m; (c) $W.e = 1.454$ N-m; and (d) $W.e = 1.944$ N-m with static load (W_s) of 12 kN.	169
3.75	Measured dynamic bending moment along instrumented pile for (a) VPG; (b) BPG subjected to coupled vibration along x direction under a static load (W_s) of 12 kN.	171
3.76	Variation of maximum dynamic bending moment for BPG-x and BPG-y subjected to coupled vibration for different excitation intensities ($W.e$) with static load (W_s) of 12 kN	172
4.1	Pile embedded in layered media and node numbering	178
4.2	Relationship between complex stiffnesses in pile coordinates, $\bar{K}$, and global stiffnesses, K , for battered pile.	187

4.3	Schematic diagram of composite medium: (a) plan view, and (b) sectional view	189
4.4	Notation for calculation of horizontal stiffness	192
4.5	Equivalent spacing of battered piles	202
4.6	Variation of (a) vertical stiffness; (b) vertical damping of single vertical pile, <i>P1</i> with frequency	210
4.7	Variation of (a) vertical stiffness; (b) vertical damping of vertical pile group, VPG with frequency	210
4.8	Comparison of variation of (a) vertical stiffness; (b) vertical damping with frequency of single batter pile, <i>P2</i> for linear approach, nonlinear approach 1 and nonlinear approach 2 for excitation intensities (<i>W.e</i>) of 0.225 N-m and 1.944 N-m.	211
4.9	Comparison of variation of (a) vertical stiffness; (b) vertical damping with frequency of single batter pile, <i>P3</i> for linear approach, nonlinear approach 1 and nonlinear approach 2 for excitation intensities (<i>W.e</i>) of 0.225 N-m and 1.944 N-m.	212
4.10	Variation of (a) vertical stiffness; (b) vertical damping of single batter pile, <i>P2</i> with frequency	213
4.11	Variation of (a) vertical stiffness; (b) vertical damping of single batter pile, <i>P3</i> with frequency	214
4.12	Variation of (a) vertical stiffness; (b) vertical damping of batter pile group, BPG with frequency	214
4.13	Comparison of variation of (a) vertical stiffness; (b) vertical damping with frequency of different single batter piles, <i>P1</i> , <i>P2</i> and <i>P3</i> for excitation intensities (<i>W.e</i>) of 0.225 N-m and 1.944 N-m.	216
4.14	Comparison of variation of (a) vertical stiffness; (b) vertical damping with frequency of different pile groups, VPG and BPG for excitation intensities (<i>W.e</i>) of 0.225 N-m and 1.944 N-m.	217
4.15	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of single vertical pile, <i>P1</i> with frequency.	220

4.16	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of vertical pile group, VPG with frequency.	222
4.17	Comparison of variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping with frequency of single batter pile, <i>P3</i> for linear approach, nonlinear approach 1 and nonlinear approach 2 for excitation intensities (<i>W.e</i>) of 0.225 N-m and 1.944 N-m.	224
4.18	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of single batter pile, <i>P2</i> with frequency.	225
4.19	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of single batter pile, <i>P3</i> with frequency.	226
4.20	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of batter pile group, BPG with frequency.	227
4.21	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of single batter pile, <i>P2</i> under coupled vibration across the pile batter angle with frequency.	228
4.22	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of single batter pile, <i>P3</i> under coupled vibration across the pile batter angle with frequency.	229
4.23	Variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping of batter pile group, BPG under coupled vibration across the pile batter angle with frequency.	230
4.24	Comparison of variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping with frequency of different single batter piles, <i>P1</i> , <i>P2</i> and <i>P3</i> for excitation intensities (<i>W.e</i>) of 0.225 N-m and 1.944 N-m.	233
4.25	Comparison of variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping with frequency of different single batter piles, <i>P1</i> , <i>P2</i> and <i>P3</i> under coupled vibrations along and across the pile batter angle for excitation intensities (<i>W.e</i>) of 1.944 N-m.	234
4.26	Comparison of variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping with frequency of different pile	236

	groups, VPG and BPG for excitation intensities ($W.e$) of 0.225 N-m and 1.944 N-m.	
4.27	Comparison of variation of (a) horizontal stiffness; (b) horizontal damping; (c) rocking stiffness; (d) rocking damping with frequency of different pile groups, VPG and BPG under coupled vibrations along and across the pile batter angle for excitation intensities ($W.e$) of 1.944 N-m.	237
4.28	Group efficiency ratio (GER) for vertical stiffness of (a) vertical pile; (b) batter pile under vertical vibration	239
4.29	Group efficiency ratio (GER) for horizontal stiffness of (a) vertical pile; (b) batter pile under coupled vibration.	240
4.30	Group efficiency ratio (GER) for rocking stiffness of (a) vertical pile; (b) batter pile under coupled vibration	240
5.1	Comparison of response curve of single vertical pile, $P1$ under vertical vibration obtained from experiments and linear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN	246
5.2	Comparison of response curve of vertical pile group, VPG under vertical vibration obtained from experiments and linear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN	246
5.3	Comparison of response curve of single batter pile, $P2$ under vertical vibration obtained from experiments and linear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	248
5.4	Comparison of response curve of single batter pile, $P3$ under vertical vibration obtained from experiments and linear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	248
5.5	Comparison of response curve of batter pile group, BPG under vertical vibration obtained from experiments and linear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	249
5.6	Zone of soil disturbance around the driven piles (a) Randolph et al. (1979); (b) Present study	253
5.7	Schematic diagram of the soil-pile model for nonlinear approach	255

5.8	Variation of boundary zone parameters of single vertical pile, <i>P1</i> for different excitation intensities (<i>W.e</i>) under vertical vibrations.	256
5.9	Variation of boundary zone parameters of vertical pile group, VPG for different excitation intensities (<i>W.e</i>) under vertical vibrations.	256
5.10	Comparison of response curve of single vertical pile, <i>P1</i> under vertical vibration obtained from experiments and nonlinear approach for different excitation intensities (<i>W.e</i>) under a static load (W_s) of 12 kN.	257
5.11	Comparison of response curve of vertical pile group, VPG under vertical vibration obtained from experiments and nonlinear approach for different excitation intensities (<i>W.e</i>) under a static load (W_s) of 12 kN.	258
5.12	Comparison of response curve of single batter pile, <i>P3</i> under vertical vibration obtained from experiments and nonlinear approach 1 for different excitation intensities (<i>W.e</i>) under a static load (W_s) of 12 kN.	260
5.13	Comparison of response curve of batter pile group, BPG under vertical vibration obtained from experiments and nonlinear approach 1 for different excitation intensities (<i>W.e</i>) under a static load (W_s) of 12 kN.	261
5.14	Variation of boundary zone parameters of single batter pile, <i>P2</i> for different excitation intensities (<i>W.e</i>) under vertical vibrations.	263
5.15	Variation of boundary zone parameters of single batter pile, <i>P3</i> for different excitation intensities (<i>W.e</i>) under vertical vibrations.	263
5.16	Variation of boundary zone parameters of batter pile group, BPG for different excitation intensities (<i>W.e</i>) under vertical vibrations.	264
5.17	Comparison of response curve of single batter pile, <i>P2</i> under vertical vibration obtained from experiments and nonlinear approach 2 for different excitation intensities (<i>W.e</i>) under a static load (W_s) of 12 kN.	264
5.18	Comparison of response curve of single batter pile, <i>P3</i> under vertical vibration obtained from experiments and nonlinear approach 2 for different excitation intensities (<i>W.e</i>) under a static load (W_s) of 12 kN.	265
5.19	Comparison of response curve of batter pile group, BPG under vertical vibration obtained from experiments and nonlinear approach 2 for different excitation intensities (<i>W.e</i>) under a static load (W_s) of 12 kN.	265

5.20	Comparison of variation of boundary zone parameters of different single batter piles, <i>P1</i> , <i>P2</i> and <i>P3</i> for excitation intensities (<i>W.e</i>) of 1.944 N-m under vertical vibrations.	268
5.21	Comparison of response curve of single batter piles, <i>P1</i> , <i>P2</i> and <i>P3</i> under vertical vibration obtained from experiments and nonlinear approach for excitation intensities (<i>W.e</i>) of 1.944 N-m under a static load (<i>W_s</i>) of 12 kN.	268
5.22	Comparison of variation of boundary zone parameters of different pile group, VPG and BPG for excitation intensities (<i>W.e</i>) of 1.944 N-m under vertical vibrations.	269
5.23	Comparison of response curve of pile groups, VPG and BPG under vertical vibration obtained from experiments and nonlinear approach for excitation intensities (<i>W.e</i>) of 1.944 N-m under a static load (<i>W_s</i>) of 12 kN.	269
5.24	Comparison of (a) horizontal; (b) rocking response curve of single vertical pile, <i>P1</i> under coupled vibration obtained from experiments and linear approach for different excitation intensities (<i>W.e</i>) under a static load (<i>W_s</i>) of 12 kN.	274
5.25	Comparison of (a) horizontal; (b) rocking response curve of vertical pile group, VPG under coupled vibration obtained from experiments and linear approach for different excitation intensities (<i>W.e</i>) under a static load (<i>W_s</i>) of 12 kN.	275
5.26	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, <i>P2</i> under coupled vibration obtained from experiments and linear approach for different excitation intensities (<i>W.e</i>) under a static load (<i>W_s</i>) of 12 kN.	275
5.27	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, <i>P3</i> under coupled vibration obtained from experiments and linear approach for different excitation intensities (<i>W.e</i>) under a static load (<i>W_s</i>) of 12 kN.	276
5.28	Comparison of (a) horizontal; (b) rocking response curve of batter pile group, BPG under coupled vibration obtained from experiments and linear	276

	approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	
5.29	Variation of boundary zone parameters of single vertical pile, $P1$ for different excitation intensities ($W.e$) under coupled vibrations.	278
5.30	Variation of boundary zone parameters of vertical pile group, VPG for different excitation intensities ($W.e$) under coupled vibrations.	279
5.31	Comparison of (a) horizontal; (b) rocking response curve of single vertical pile, $P1$ under coupled vibration obtained from experiments and nonlinear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	280
5.32	Comparison of (a) horizontal; (b) rocking response curve of vertical pile group, VPG under coupled vibration obtained from experiments and nonlinear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	281
5.33	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, $P3$ under coupled vibration obtained from experiments and nonlinear approach 1 for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	283
5.34	Variation of boundary zone parameters of single batter pile, $P2$ for different excitation intensities ($W.e$) under coupled vibrations.	285
5.35	Variation of boundary zone parameters of single batter pile, $P3$ for different excitation intensities ($W.e$) under coupled vibrations.	286
5.36	Variation of boundary zone parameters of batter pile group, BPG for different excitation intensities ($W.e$) under coupled vibrations.	286
5.37	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, $P2$ under coupled vibration obtained from experiments and nonlinear approach 2 for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	287
5.38	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, $P3$ under coupled vibration obtained from experiments and nonlinear	287

	approach 2 for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	
5.39	Comparison of (a) horizontal; (b) rocking response curve of batter pile group, BPG under coupled vibration obtained from experiments and nonlinear approach 2 for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	288
5.40	Comparison of variation of boundary zone parameters of different single batter piles, $P1$, $P2$ and $P3$ for excitation intensities ($W.e$) of 1.944 N-m under coupled vibrations.	290
5.41	Comparison of (a) horizontal; (b) rocking response curve of single batter piles, $P1$, $P2$ and $P3$ under coupled vibration obtained from experiments and nonlinear approach for excitation intensities ($W.e$) of 1.944 N-m under a static load (W_s) of 12 kN.	291
5.42	Comparison of variation of boundary zone parameters of different pile groups, VPG and BPG for excitation intensity ($W.e$) of 1.944 N-m under coupled vibrations.	291
5.43	Comparison of (a) horizontal; (b) rocking response curve of different pile groups, VPG and BPG under coupled vibration obtained from experiments and nonlinear approach for excitation intensities ($W.e$) of 1.944 N-m under a static load (W_s) of 12 kN.	292
5.44	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, $P3$ under coupled vibration across the pile batter angle obtained from experiments and linear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	294
5.45	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, $P2$ under coupled vibration across the pile batter angle obtained from experiments and nonlinear approach 1 for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	295
5.46	Comparison of (a) horizontal; (b) rocking response curve of single batter pile, $P3$ under coupled vibration across the pile batter angle obtained from	296

	experiments and nonlinear approach 1 for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	
5.47	Comparison of (a) horizontal; (b) rocking response curve of batter pile group, BPG under coupled vibration across the pile batter angle obtained from experiments and nonlinear approach 1 for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	296
5.48	Comparison of (a) horizontal; (b) rocking response curve of single batter piles, $P2$ and $P3$ under coupled vibration along and across the pile batter angle obtained from experiments and nonlinear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	298
5.49	Comparison of (a) horizontal; (b) rocking response curve of pile groups, VPG and BPG under coupled vibration along and across the pile batter angle obtained from experiments and nonlinear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	299
5.50	Comparison of variation of dynamic axial load with depth for (a) single vertical pile, $P1$; (b) vertical pile group, VPG obtained from experiments and nonlinear approach under vertical vibrations for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	306
5.51	Comparison of variation of dynamic axial load with depth for (a) single batter pile, $P2$; (b) single batter pile, $P3$; (c) batter pile group, BPG obtained from experiments and nonlinear approach for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	307
5.52	Comparison of variation of dynamic bending moment with depth for (a) single vertical pile, $P1$; (b) single batter pile, $P2$; (c) single batter pile, $P3$ obtained from experiments and nonlinear approach under coupled vibrations for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	308
5.53	Comparison of variation of dynamic bending moment with depth for (a) vertical pile group, VPG; (b) batter pile group, BPG obtained from experiments and nonlinear approach under coupled vibrations for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	309

5.54	Comparison of variation of dynamic axial load with depth for (a) single batter pile, $P2$; (b) single batter pile, $P3$; (c) batter pile group, BPG obtained from experiments and nonlinear approach under coupled vibrations across the pile batter angle for different excitation intensities ($W.e$) under a static load (W_s) of 12 kN.	309
5.55	Variation of soil-pile separation lengths of single piles, $P1$, $P2$ and $P3$ with excitation intensities ($W.e$) for vertical vibrations	314
5.56	Variation of soil-pile separation lengths of single piles, $P1$, $P2$ and $P3$ with excitation intensities ($W.e$) for coupled vibrations	314
5.57	Variation of soil-pile separation lengths of 2 x 2 pile group, VPG and BPG with excitation intensities ($W.e$) for vertical vibrations	315
5.58	Variation of soil-pile separation lengths of 2 x 2 pile group, VPG and BPG with excitation intensities ($W.e$) for coupled vibrations	315
5.59	Variation of soil-pile separation lengths of single piles, $P1$, $P2$ and $P3$ with maximum resonant amplitude under vertical vibrations	316
5.60	Variation of soil-pile separation lengths of single piles, $P1$, $P2$ and $P3$ with maximum horizontal resonant amplitude under coupled vibrations	316
5.61	Variation of soil-pile separation lengths of 2 x 2 pile groups, VPG and BPG with maximum resonant amplitude under vertical vibrations	317
5.62	Variation of soil-pile separation lengths of 2 x 2 pile groups, VPG and BPG with maximum horizontal resonant amplitude under coupled vibrations	317
6.1	Variation of boundary zone parameters of single vertical pile, $P1$ ($\beta = 0^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	325
6.2	Variation of boundary zone parameters of single batter pile, $P2$ ($\beta = 10^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	326
6.3	Variation of boundary zone parameters of single batter pile, $P3$ ($\beta = 20^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	327

6.4	Variation of boundary zone parameters of 2×2 vertical pile group, VPG ($\beta = 0^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	328
6.5	Variation of boundary zone parameters of 2×2 batter pile group, BPG ($\beta = 20^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	329
6.6	Variation of boundary zone parameters of single vertical pile, $P1$ ($\beta = 0^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	332
6.7	Variation of boundary zone parameters of single batter pile, $P2$ ($\beta = 10^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	333
6.8	Variation of boundary zone parameters of single batter pile, $P3$ ($\beta = 20^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	334
6.9	Variation of boundary zone parameters of 2×2 vertical pile group, VPG ($\beta = 0^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	335
6.10	Variation of boundary zone parameters of 2×2 batter pile group, BPG ($\beta = 20^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	336
6.11	Comparison of the frequency versus amplitude responses of the prototype single piles obtained from the scaled experimental responses and theoretical analysis (nonlinear approach) results under vertical vibration (a) single vertical pile, $P1$ ($\beta = 0^\circ$) (b) single batter pile, $P2$ ($\beta = 10^\circ$) (c) single batter pile, $P3$ ($\beta = 20^\circ$)	347
6.12	Comparison of the frequency versus amplitude responses of the prototype single vertical pile, $P1$ ($\beta = 0^\circ$) obtained from the scaled experimental responses and theoretical analysis (nonlinear approach) results under coupled vibration (a) horizontal mode (b) rocking mode	348

6.13	Comparison of the frequency versus amplitude responses of the prototype single batter pile, $P2$ ($\beta = 10^\circ$) obtained from the scaled experimental responses and theoretical analysis (nonlinear approach 2) results under coupled vibration (a) horizontal mode (b) rocking mode	348
6.14	Comparison of the frequency versus amplitude responses of the prototype single batter pile, $P3$ ($\beta = 20^\circ$) obtained from the scaled experimental responses and theoretical analysis (nonlinear approach 2) results under coupled vibration (a) horizontal mode (b) rocking mode	349
6.15	Comparison of frequency versus amplitude responses of the single pile obtained from the vertical vibration tests (Manna and Baidya 2010b) and nonlinear approach using design charts	359
6.16	Comparison of frequency versus amplitude responses of the 2×2 vertical pile group obtained from the vertical vibration tests (Manna and Baidya 2009b) and nonlinear approach using design charts	359
6.17	Comparison of frequency versus amplitude responses of the 2×2 batter pile group with pile batter angle of 20 degrees obtained from the lateral (horizontal) vibration tests along the pile batter angle (Bharathi et al. 2018) and nonlinear approach 2 using design charts	360
6.18	Pile and soil properties	361
6.19	Variation of frequency-amplitude response of model-scaled ($l = 3\text{m}$, $d = 0.114\text{m}$) with eccentric moments of 0.225 N-m subjected to vertical vibrations	365
6.20	Variation of frequency-amplitude response of design example for pile ($l = 18\text{m}$, $d = 0.68\text{m}$) with eccentric moments of 714 N-m subjected to vertical vibrations	383

LIST OF TABLES

Table No.	Description	Page No.
2.1	Discussion on theoretical and experimental studies	51
3.1	Properties of soil with depth	62
3.2	Properties of the test piles	63
3.3	Dynamic testing of piles	80
3.4	Details of pile cap, steel plates and oscillator under vertical vibrations	82
3.5	Experimental resonant frequency and amplitude values of single vertical and batter piles with static load (W_s) of 12 kN and 14 kN under vertical vibration	101
3.6	Soil-pile separation length obtained from vertical vibration tests under a static load (W_s) of 12 kN under vertical vibration	114
3.7	Experimental values of resonant frequencies and amplitudes of single vertical and batter piles subjected to coupled vibration along (x direction) the pile batter angle (β) under static load of 12 kN and 14 kN	124
3.8	Soil-pile separation length (l_s) obtained from coupled vibration tests subjected to coupled vibration along the pile batter angle under a static load (W_s) of 12 kN	135
3.9	Experimental resonant frequency and resonant amplitude values of vertical pile group and batter pile group under vertical vibrations	146
3.10	Experimental values of resonant frequencies and amplitudes of vertical pile group (VPG) and batter pile	161

	group (BPG) subjected to coupled vibration along x direction under a static load of 12 kN and 14 kN	
3.11	Soil-pile separation length (l_s) of VPG and BPG subjected to coupled vibration along x direction under a static load (W_s) of 12 kN	170
5.1	Properties of the pile	244
5.2	The boundary zone parameters of soil used in the theoretical analysis	251
5.3	Separation lengths obtained from the dynamic field tests and separation lengths used in the nonlinear approach for single vertical pile, $P1$ and vertical pile group, VPG for different excitation intensities ($W.e$) under a static load of 12 kN for vertical vibration	270
5.4	Separation lengths obtained from the dynamic field tests and separation lengths used in the nonlinear approach for single batter piles, $P2$ and $P3$ and batter pile group, BPG for different <i>excitation intensities</i> ($W.e$) under a static load of 12 kN for vertical vibration	270
5.5	Comparison of experimental and theoretical results of single piles, $P1$, $P2$ and $P3$ for vertical vibration	271
5.6	Comparison of experimental and theoretical results of pile group, VPG and BPG for vertical vibration	272
5.7	Separation lengths obtained from the dynamic field tests and separation lengths used in the nonlinear approach for single vertical pile, $P1$ and vertical pile group, VPG for different excitation intensities ($W.e$) under a static load of 12 kN for coupled vibration	299
5.8	Separation lengths obtained from the dynamic field tests and separation lengths used in the nonlinear approach for single batter piles, $P2$ and $P3$ and batter pile group, BPG	300

	for different excitation intensities ($W.e$) under a static load of 12 kN for coupled vibration	
5.9	Comparison of experimental and theoretical results of single piles, $P1$, $P2$ and $P3$ for coupled vibration under static load (W_s) of 12 kN	301
5.10	Comparison of experimental and theoretical results of single piles, $P1$, $P2$ and $P3$ for coupled vibration under static load (W_s) of 14 kN	302
5.11	Comparison of experimental and theoretical results of pile groups, VPG and BPG for coupled vibration under static load (W_s) of 12 kN	303
5.12	Comparison of experimental and theoretical results of pile groups, VPG and BPG for coupled vibration under static load (W_s) of 14 kN	304
6.1	Coefficient of determination for different depths for plot between boundary zone parameters of single vertical pile, $P1$ ($\beta = 0^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	337
6.2	Coefficient of determination for different depths for variation of boundary zone parameters of single batter pile, $P2$ ($\beta = 10^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	337
6.3	Coefficient of determination for different depths for variation of boundary zone parameters of single batter pile, $P3$ ($\beta = 20^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	338

6.4	Coefficient of determination for different depths for variation of boundary zone parameters of 2×2 vertical pile group, VPG ($\beta = 0^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	338
6.5	Coefficient of determination for different depths for variation of boundary zone parameters of 2×2 batter pile group, BPG ($\beta = 20^\circ$) with excitation intensities for vertical vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	339
6.6	Coefficient of determination for different depths for variation of boundary zone parameters of single vertical pile, $P1$ ($\beta = 0^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	339
6.7	Coefficient of determination for different depths for variation of boundary zone parameters of single batter pile, $P2$ ($\beta = 10^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	340
6.8	Coefficient of determination for different depths for variation of boundary zone parameters of single batter pile, $P3$ ($\beta = 20^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	340
6.9	Coefficient of determination for different depths for variation of boundary zone parameters of 2×2 vertical pile group, VPG ($\beta = 0^\circ$) with excitation intensities for coupled	341

	vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	
6.10	Coefficient of determination for different depths for variation of boundary zone parameters of 2×2 batter pile group, BPG ($\beta = 20^\circ$) with excitation intensities for coupled vibration (a) shear modulus reduction ratio (b) weak zone soil damping (c) thickness ratio	341
6.11	Pile scaling law and factors	343
6.12	Parameters of the prototype piles using scaling law	344
6.13	Comparison of resonant frequency and resonant amplitude values of prototype piles, $P1$ ($\beta = 0^\circ$), $P2$ ($\beta = 10^\circ$) and $P3$ ($\beta = 20^\circ$) for different pile lengths of 19.8 m, 23.1 m and 26.4 m obtained from scaled experiment and theoretical analysis under vertical vibration	350
6.14	Comparison of resonant frequency and resonant amplitude values of prototype pile groups, VPG ($\beta = 0^\circ$) and BPG ($\beta = 20^\circ$) for different pile lengths of 19.8 m, 23.1 m and 26.4 m obtained from scaled experiment and theoretical analysis under vertical vibration	351
6.15	Comparison of resonant frequency and resonant amplitude values of prototype piles, $P1$ ($\beta = 0^\circ$), $P2$ ($\beta = 10^\circ$) and $P3$ ($\beta = 20^\circ$) of 19.8 m length obtained from scaled experiment and theoretical analysis under coupled vibration	352
6.16	Comparison of resonant frequency and resonant amplitude values of prototype piles, $P1$ ($\beta = 0^\circ$), $P2$ ($\beta = 10^\circ$) and $P3$ ($\beta = 20^\circ$) of 23.1 m length obtained from scaled experiment and theoretical analysis under coupled vibration	353
6.17	Comparison of resonant frequency and resonant amplitude values of prototype piles, $P1$ ($\beta = 0^\circ$), $P2$ ($\beta = 10^\circ$) and $P3$ ($\beta = 20^\circ$) of 26.4 m length obtained from scaled experiment and theoretical analysis under coupled vibration	354

6.18	Comparison of resonant frequency and resonant amplitude values of prototype pile groups, VPG ($\beta = 0^\circ$) and BPG ($\beta = 20^\circ$) of 19.8 m obtained from scaled experiment and theoretical analysis under coupled vibration	355
6.19	Comparison of resonant frequency and resonant amplitude values of prototype pile groups, VPG ($\beta = 0^\circ$) and BPG ($\beta = 20^\circ$) of 23.1 m obtained from scaled experiment and theoretical analysis under coupled vibration	356
6.20	Comparison of resonant frequency and resonant amplitude values of prototype pile groups, VPG ($\beta = 0^\circ$) and BPG ($\beta = 20^\circ$) of 26.4 m obtained from scaled experiment and theoretical analysis under coupled vibration	357
6.21	Scaling down the soil parameters, pile parameters and machine parameters	364
6.22	Weak boundary zone parameters for eccentric moment of 0.225 N-m (model scaled value) for pile length of 3 m.	365

LIST OF SYMBOLS

Most of the symbols are defined as they appear in the thesis. Some of the most common symbols, which are used repeatedly, listed below.

Roman symbols

a_o	Dimensionless frequency
$\overline{a_o}$	Dimensionless frequency for composite medium
A	Cross-sectional area of the pile
A_1	Area of the topmost element of the pile
A_H	Amplitude of horizontal motion
A_{H1-res}, A_{H2-res}	First and second resonant amplitudes for horizontal motion
A_s	Steady state amplitude
A_V	Amplitude of vertical motion
A_{V-res}	Resonant amplitude for vertical motion
b	Pile radius plus thickness of inner soil medium ($= R + t_m$)
c	Coefficient of pile internal damping
c_u^c	Horizontal damping of embedded footing or pile cap
c_{uu}^1	Horizontal damping constant of single pile
$c_{u\psi}^c$	Cross damping of embedded footing or pile cap
$c_{u\psi}^1 = c_{\psi u}^1$	Cross damping constant of single pile
c_w^c	Vertical damping of embedded footing or pile cap
c_{ww}^1	Vertical damping constant of single pile
c_{ψ}^c	Rocking damping of embedded footing or pile cap
$c_{\psi\psi}^1$	Rocking damping constant of single pile
C_{cu1}, C_{cu2}	Stiffness and damping parameters of half-space for horizontal vibration
C_{cw1}, C_{cw2}	Stiffness and damping parameters of half-space for vertical vibration

$C_{c\psi 1}, C_{c\psi 2}$	Stiffness and damping parameters of half-space for rocking vibration
C_{uu}	Total horizontal damping constants of pile
$C_{u\psi}$	Total cross damping constants of pile
$C_{\psi\psi}$	Total rocking damping constants of pile
d	Diameter of the pile
D	Damping ratio
D_i	Damping ratio corresponding to frequency of vibration ω_i
D_s	Dimensionless damping constant
D_{sm}	Dimensionless damping constant of inner medium
e	Eccentric distance of the rotating masses
E_b	Young's modulus of the bearing stratum
E_p	Young's modulus of the pile
E_s	Young's modulus of the soil along the pile
f_{c1}, f_{c2}	Dimensionless cross stiffness and damping parameters
f_{mc1}, f_{mc2}	Dimensionless cross stiffness and damping constants of the composite medium
f_{mu1}, f_{mu2}	Dimensionless horizontal stiffness and damping constants of the composite medium
f_{mw1}, f_{mw2}	Dimensionless vertical stiffness and damping constants of the composite medium
$f_{m\psi 1}, f_{m\psi 2}$	Dimensionless rocking stiffness and damping constants of the composite medium
f_n	Resonant frequency for vertical vibration
f_{n1}, f_{n2}	First and second resonant frequencies for coupled vibration
f_{u1}, f_{u2}	Dimensionless horizontal stiffness and damping parameters
f_{w1}, f_{w2}	Dimensionless vertical stiffness and damping parameters
$f_{\psi 1}, f_{\psi 2}$	Dimensionless rocking stiffness and damping parameters
f_{st}	Static flexibility of a single pile
$[f]$	Group flexibility matrix
$F(A)$	Restoring force
F_i	Dimensionless function

g	Acceleration due to gravity
G	Shear modulus of soil
G'	Imaginary part of the complex shear modulus of soil
G^*	Complex shear modulus of outer soil medium
G_{cb}	Dynamic shear modulus of soil below the footing
G_{cs}	Dynamic shear modulus of the side layers of footing
G_m	Shear modulus of inner soil medium
G_m^*	Complex shear modulus of inner weak soil medium
h	Embedded depth of footing or pile cap
H	Height (length) of the element
H_1, H_2	Horizontal end forces of element corresponding to end displacements
i	Complex number ($i = \sqrt{-1}$)
I	Moment of inertia of pile cross section
I_1	Moment of inertia of the topmost element of the pile
I_{ψ}	Mass moment of inertia about a horizontal axis passing through the center of gravity
k_{um}	Horizontal stiffness of the composite medium
k_{us}	Complex horizontal stiffness of soil per unit length of pile
k_{ut}	Complex horizontal stiffness of soil at the pile tip
k_{wm}	Vertical stiffness of the composite medium
k_{ws}	Complex vertical stiffness of soil per unit length of pile
k_{wt}	Complex vertical stiffness of soil at the pile tip
$k_{\psi m}$	Rocking stiffness of the composite medium
$k_{\psi s}$	Complex rocking stiffness of soil per unit length of pile
$k_{\psi t}$	Complex rocking stiffness of soil at the pile tip
$\bar{k}_u$	Horizontal static stiffness of single pile
$k_u^{(1)}$	The soil stiffness of the outer medium for horizontal vibration
$k_u^{(2)}$	The soil stiffness of the inner medium for horizontal vibration
k_u^c	Horizontal stiffness of embedded footing or pile cap

k_{uu}^1	Horizontal stiffness of single pile
$k_{u\psi}^1 = k_{\psi u}^1$	Cross stiffness of single pile
$k_{u\psi}^c$	Cross stiffness of embedded footing or pile cap
$\bar{k}_w$	Vertical static stiffness of single pile
k_w^c	Vertical stiffness of embedded footing or pile cap
k_{ww}^1	Vertical stiffness of single pile
$k_{\psi\psi}^c$	Rocking stiffness of embedded footing or pile cap
$k_{\psi\psi}^1$	Rocking stiffness of single pile
K_0	Modified Bessel functions of order zero
K_1	Modified Bessel functions of order one
K^G	Complex group stiffness
$K_e(A)$	Equivalent linear stiffness
K_{ij}	Elements of the structure matrix
K_{uu}	Total horizontal stiffness of pile
$K_{u\psi}$	Total cross stiffness of pile
K_{ww}	Total vertical stiffness of pile
$K_{\psi\psi}$	Total rocking stiffness of pile
K_{mu}^1	Total complex horizontal stiffness of single pile in composite medium
K_{mw}^1	Total complex vertical stiffness of single pile in composite medium
$K_{m\psi}^1$	Total complex rocking stiffness of single pile in composite medium
$K_{mu\psi}^1 = K_{m\psi u}^1$	Total complex cross stiffness of single pile in composite medium
K_{uu}^1	Complete horizontal stiffness of single pile
$K_{u\psi}^1 = K_{\psi u}^1$	Complete cross stiffness of single pile
K_{ww}^1	Complete vertical stiffness of single pile

$K_{\psi\psi}^1$	Complete rocking stiffness of single pile
K_u^G	Horizontal stiffness of pile group
$K_{u\psi}^G$	Cross stiffness of pile group
K_w^G	Vertical stiffness of pile group
K_{ψ}^G	Rocking stiffness of pile group
$[k_t]$	Element stiffness matrix of the soil underlying the pile tip
$[k_u]$	Element stiffness matrix for horizontal vibration
$[k_w]$	Element stiffness matrix for vertical vibration
$[K]$	Structure stiffness matrix
$[K_u]$	Structure stiffness matrix of single pile for horizontal vibration
$[K_w]$	Structure stiffness matrix of single pile for vertical vibration
l_s	Separation length between pile and soil
L	Length of the pile
m	Mass of eccentric rotating part in oscillator
m_{eff}	Effective mass of the system
m_s	Total mass of footing or pile cap
M	Real amplitude of moment
M_1, M_2	Moment of element corresponding to end rotations
$M(t)$	Total moment of excitation for coupled vibration
$M_e(t)$	Excitation moment
n	Number of piles in a group
N	SPT value
N_1, N_2	Vertical end forces of element corresponding to end displacements
N_{st}	Static axial force (load of the pile)
$N_x(t)$	Resultant horizontal reaction acting on embedded surface (sides) of footing
$N_{\psi}(t)$	Reactive moment of forces acting on footing sides about center of gravity

p	Lateral soil resistance
p_u	Ultimate soil resistance
$P1$	Single vertical pile with pile batter angle of 0 degrees
$P2$	Single batter pile with pile batter angle of 10 degrees
$P3$	Single batter pile with pile batter angle of 20 degrees
$P(t)$	Horizontal excitation force
$\{P\}$	Vector of forces
r_1	Radius of the topmost element of the pile
r_c	Radius of footing or pile cap
R	Radius of circular pile
R_{inter}	strength reduction factor
$R_x(t)$	Horizontal reaction at the footing base
$R_\psi(t)$	Reactive moment of forces acting at footing base about center of gravity
s	Spacing of piles in a group
s_u	Undrained shear strength
S_{cu1}, S_{cu2}	Stiffness and damping parameters of side layer for horizontal vibration
S_{cw1}, S_{cw2}	Stiffness and damping parameters of side layer for vertical vibration
$S_{c\psi1}, S_{c\psi2}$	Stiffness and damping parameters of side layer for rocking vibration
S_{ou1}, S_{ou2}	Horizontal stiffness and damping parameters of outer medium
S_{mu1}, S_{mu2}	Dimensionless stiffness and damping parameters of composite medium for horizontal vibration
S_{mw1}, S_{mw2}	Dimensionless stiffness and damping parameters of composite medium for vertical vibration
$S_{m\psi1}, S_{m\psi2}$	Dimensionless stiffness and damping parameters of composite medium for rocking
S_{u1}, S_{u2}	Real and imaginary parts of the dimensionless complex stiffness in horizontal direction (u)

S_{w1}, S_{w2}	Real and imaginary parts of the dimensionless complex stiffness in vertical direction (w)
$S_{\psi1}, S_{\psi2}$	Real and imaginary parts of the dimensionless complex stiffness in rotation or rocking (ψ)
t	Time
t_m	Thickness of inner soil medium
u	Horizontal displacement of pile
u_1, u_2	Horizontal end displacements of the element
VPG	Vertical pile group
BPG	Batter pile group
V_{La}	Lysmer's analog velocity
V_n	Shear wave velocity of soil in the lowest layer
V_s	Shear wave velocity of the soil
w	Vertical displacement of pile
w_1, w_2	Vertical end displacements of the element
w_{res}	Resonant amplitude of vertical vibration
W	Weight of eccentric rotating part in oscillator
W_s	Total weight of footing or total static load on pile
x_i, x_j	Distances of each pile from centre of gravity of foundation
y	Pile deflection
y_c	Critical pile deflection
z_c	Height of the center of gravity above the footing base
z_e	Height of horizontal excitation force above the center of gravity of footing

Greek symbols

α	Complex dynamic interaction factor
α_1	Real part of the complex dynamic interaction factor
α_2	Imaginary part of the complex dynamic interaction factor
β	Pile batter angle

α_{ij}	Dynamic interaction factor of pile group
α_{st}	Static interaction factor
$[\alpha]$	Matrix of interaction factors
$[\alpha]_H$	Interaction matrix for horizontal translation and rotations
$[\alpha]_V$	Interaction matrix for vertical translation
β	Frequency-independent soil material damping
δ	Loss angle
δ_r	Relative embedment
δ_{dij}	Dynamic displacement of pile i due to load on pile j
δ_{sij}	Static displacement of pile j due to its own load
$\{\delta\}$	Vector of displacements
θ	Angle of eccentricity
ϕ	Friction angle of soil
φ	Angle between the direction of load and the plane in which piles lie
ω	Circular frequency
ω_1, ω_2	The frequencies corresponding to the points of interaction between the response curve and a line passing through the origin of coordinates
ω_i	The components of the rotational vector
ω_m	Damped resonant circular frequency
ω_n	Undamped resonant circular frequency
ω_{n1}, ω_{n2}	Two resonant frequencies of coupled vibration
μ	Mass of the pile per unit length
ρ	Mass density
ν	Poisson's ratio of soil
ν_m	Poisson's ratio of inner zone
ν_s	Poisson's ratio of outer zone
λ, λ'	Real part and imaginary part of Lamé's constant
ξ	Effective mass coefficient

ψ	Rotation (rocking) in vertical plane
ψ_1, ψ_2	Rotational (rocking) end displacements of the element
ψ_r	Amplitude of rocking motion
$\psi_{r1-res}, \psi_{r2-res}$	First and second resonant amplitudes for rocking motion
$\mathcal{E}_{2i,2j}^H$	Complex elements of $[\alpha]_H^{-1}$ that correspond to the moments associated with rotations
$\mathcal{E}_{2i-1,2j-1}^H$	Complex elements of $[\alpha]_H^{-1}$ that corresponds to the horizontal forces associated with horizontal displacements
$\mathcal{E}_{ij}^V$	Elements of $[\alpha]_V^{-1}$
Λ	Complex frequency parameter
Δ	Relative volume change
Δ_h	Maximum amplitude of pile for horizontal vibration
Δ_r	Maximum amplitude of pile for rocking vibration
Δ_v	Maximum amplitudes of pile for vertical motion
Ω	Backbone curve

Abbreviations

BEM	Boundary element method
BH	Bore hole
C.G.	Center of gravity
DC	Direct-current
Deg	Degree
DOF	Degree of freedom
Eq.	Equation
FE	Finite element
FEM	Finite element method
Fig.	Figure
GER	Group efficiency ratio
Hz	Hertz

IS	Indian standard
kN	Kilo Newton
L.L.	Liquid limit
m	Meter
mm	Millimeter
M.P.F.	Mass participation factor
P.L.	Plastic limit
Rad	Radian
RPM	Revolutions per minute
SDOF	Single degree of freedom
SPT	Standard penetration test
USCS	Unified soil classification system
2-D	Two dimensional
3-D	Three dimensional