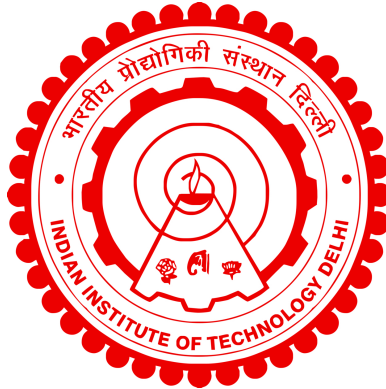


**INVERSE PROBLEMS IN  
THERMO-FLUID SYSTEMS USING  
POLYNOMIAL CHAOS EXPANSIONS  
AND BAYESIAN INFERENCE**

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**DEPARTMENT OF MECHANICAL  
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**INDIAN INSTITUTE OF TECHNOLOGY  
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**OCTOBER 2022**



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By

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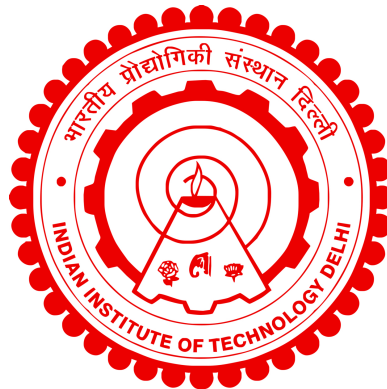
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# Certificate

This is to certify that the thesis entitled “**Inverse problems in thermo-fluid systems using polynomial chaos expansions and Bayesian inference**”, being submitted by, **Ms. Sufia Khatoon** to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of original research work carried out by her under our supervision in conformity with the rules and regulations of the institute. The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any Degree or Diploma.

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**Sufia Khatoon**

# Abstract

Inverse problems find their applications in engineering of various thermo-fluid systems. Finding a solution to inverse problems has been challenging due to their ill-posed nature. A solution to inverse problems is a promising area of research that could help find various unknown model parameters that are otherwise difficult to obtain. In the past few decades, various deterministic and stochastic methods have been developed to find a solution to inverse problems. The present work aims to develop a computational framework using Bayesian inference, which leads to forward propagation of uncertainty in various thermo-fluid model problems and solves the corresponding inverse problems.

To solve inverse problems in various thermo-fluid systems, a computational framework has been developed using Bayesian inference. The framework leverages the polynomial chaos expansions (PCEs) to generate computationally efficient and statistically equivalent surrogate model of the computationally expensive forward model. The applicability of the framework is established by validating different model problems in thermo-fluid systems.

At first, the developed framework is used to find various non-dimensional parameters in a heat and mass transfer problem in porous media. We solved an inverse problem in a coupled heat and mass transfer system to estimate model parameters in a one-dimensional capillary porous media using Luikov's model. These solutions help to obtain various thermophysical properties of the porous medium.

Next, we demonstrate the application of the framework to heat conduction models. We describe the accelerated Bayesian inference framework to solve inverse heat conduction problems to estimate boundary heat flux. Along with the polynomial chaos expansions to provide a computationally efficient and statistically equivalent surrogate model for the computationally expensive forward model, this model problem also includes dimensional-

ity reduction using the Karhunen-Loeve (K-L) expansion. We demonstrate the potential of this approach using three model problems to estimate transient heat flux in inverse heat conduction models. We represent the heat flux using K-L expansion to reduce the high dimensionality of the heat flux and compute the posterior probability distribution of the heat flux using the temperature measurements. The first inverse problem involves the estimation of time-varying heat flux in a one-dimensional slab using transient temperature measurements. The second problem consists of estimating the unknown transient heat flux of the disc in a realistic disc brake system. The third problem involves the estimation of the surface heat flux on a sounding rocket having a realistic three-dimensional geometry of the rocket module used in actual flight tests.

Lastly, we have demonstrated the application of the framework on large-scale petroleum reservoir models. We estimated the various geological properties of a petroleum reservoir which helps in optimising reservoir performance and risk analysis. We studied the effect of uncertain geological parameters such as porosity and permeability on simulation outputs, such as oil production rate and gas production rate, using the Bayesian inference-based non-intrusive polynomial chaos method. Our simulations yield the mean and standard deviation of production forecasts and also determine the relative contribution of uncertainty in input parameters on the predictions. We performed simulations of petroleum reservoirs, considering the heterogeneity of the reservoir. We used the developed stochastic technique based on PC expansions to propagate uncertainty from model parameters to model predictions. The developed framework can also be used along with commercial simulators of petroleum reservoirs.

## सार

व्युत्क्रम समस्याएं विभिन्न थर्मो-तरल प्रणालियों की इंजीनियरिंग में उनके अनुप्रयोगों का पता लगाती हैं। विपरीत समस्याओं का समाधान खोजना उनके गलत स्वभाव के कारण चुनौतीपूर्ण रहा है। व्युत्क्रम समस्याओं का समाधान अनुसंधान का एक आशाजनक क्षेत्र है जो विभिन्न प्रकार की समस्याओं को खोजने में मदद कर सकता है अज्ञात मॉडल पैरामीटर जिन्हें प्राप्त करना अन्यथा कठिन होता है। पिछले कुछ दशकों में, इसका समाधान खोजने के लिए विभिन्न नियतात्मक और स्टोकेस्टिक विधियों का विकास किया गया है व्युत्क्रम समस्याएं। वर्तमान कार्य का उद्देश्य का उपयोग करके एक कम्प्यूटेशनल ढांचा विकसित करना है बायेंसियन अनुमान, जो विभिन्न थर्मोफ्लुइड में अनिश्चितता के आगे प्रसार की ओर जाता है मॉडल की समस्याओं को हल करता है और संबंधित प्रतिलोम समस्याओं को हल करता है।

विभिन्न थर्मो-तरल प्रणालियों में व्युत्क्रम समस्याओं को हल करने के लिए, एक कम्प्यूटेशनल ढांचा बायेंसियन अनुमान का उपयोग करके विकसित किया गया है। ढांचा बहुपद का लाभ उठाता है कम्प्यूटेशनल रूप से कुशल और सांख्यिकीय रूप से समकक्ष उत्पन्न करने के लिए अराजकता विस्तार (पीसीई) कम्प्यूटेशनल रूप से महंगे फॉरवर्ड मॉडल का सरोगेट मॉडल। की प्रयोज्यता थर्मो-फ्लुइड सिस्टम में विभिन्न मॉडल समस्याओं को मान्य करके ढांचा स्थापित किया गया है।

सबसे पहले, विकसित ढांचे का उपयोग विभिन्न गैर-आयामी मापदंडों को खोजने के लिए किया जाता है झरझरा मीडिया में एक गर्मी और बड़े पैमाने पर स्थानांतरण समस्या में। हमने एक उलटा समस्या हल की एक आयामी में मॉडल मापदंडों का अनुमान लगाने के लिए एक युग्मित गर्मी और द्रव्यमान हस्तांतरण प्रणाली में लुइकोव के मॉडल का उपयोग करते हुए केशिका झरझरा मीडिया। ये समाधान प्राप्त करने में मदद करते हैं झरझरा माध्यम के विभिन्न थर्मोफिजिकल गुण।

इसके बाद, हम चालन मॉडल को गर्म करने के लिए ढांचे के अनुप्रयोग को प्रदर्शित करते हैं। हम उलटा गर्मी चालन को हल करने के लिए त्वरित बायेंसियन अनुमान ढांचे का वर्णन करते हैं सीमा गर्मी प्रवाह का अनुमान लगाने में समस्याएं। बहुपद अराजकता के विस्तार के साथ के लिए एक कम्प्यूटेशनल रूप से कुशल और सांख्यिकीय रूप से समकक्ष सरोगेट मॉडल प्रदान करने के लिए कम्प्यूटेशनल रूप से महंगा फॉरवर्ड मॉडल, इस मॉडल समस्या में आयामीता भी शामिल है Karhunen-Loeve (K-L) विस्तार का उपयोग करके ity में कमी। हम क्षमता प्रदर्शित करते हैं

प्रतिलोम में क्षणिक ऊष्मा प्रवाह का अनुमान लगाने के लिए तीन मॉडल समस्याओं का उपयोग करते हुए इस दृष्टिकोण का गर्मी चालन मॉडल। हम कम करने के लिए K-L विस्तार का उपयोग करके ऊष्मा प्रवाह का प्रतिनिधित्व करते हैं गर्मी प्रवाह की उच्च आयामीता और पश्च संभाव्यता वितरण की गणना करें तापमान माप का उपयोग करके गर्मी प्रवाह का। पहली उलटा समस्या शामिल है क्षणिक तापमान का उपयोग करके एक-आयामी स्लैब में समय-भिन्न गर्मी प्रवाह का अनुमान माप। दूसरी समस्या अज्ञात क्षणिक का अनुमान लगाने की है यथार्थवादी डिस्क ब्रेक सिस्टम में डिस्क का ताप प्रवाह। तीसरी समस्या में अनुमान शामिल है एक यथार्थवादी त्रि-आयामी होने वाले ध्वनि वाले रॉकेट पर सतह के ताप प्रवाह का वास्तविक उड़ान परीक्षणों में प्रयुक्त रॉकेट मॉड्यूल की ज्यामिति।

अंत में, हमने बड़े पैमाने पर पेट्रोलियम पर ढांचे के अनुप्रयोग का प्रदर्शन किया है जलाशय मॉडल। हमने पेट्रोलियम जलाशय के विभिन्न भूवैज्ञानिक गुणों का अनुमान लगाया जो जलाशय के प्रदर्शन और जोखिम विश्लेषण को अनुकूलित करने में मदद करता है। हमने के प्रभाव का अध्ययन किया सिमुलेशन आउटपुट पर सरंधता और पारगम्यता जैसे अनिश्चित भूवैज्ञानिक पैरामीटर, जैसे कि तेल उत्पादन दर और गैस उत्पादन दर, बायेसियन अनुमान-आधारित . का उपयोग करते हुए गैर-घुसपैठ बहुपद अराजकता विधि। हमारे सिमुलेशन से माध्य और मानक प्राप्त होते हैं उत्पादन पूर्वानुमानों का विचलन और अनिश्चितता के सापेक्ष योगदान को भी निर्धारित करते हैं भविष्यवाणियों पर इनपुट मापदंडों में। हमने पेट्रोलियम का अनुकरण किया जलाशय, जलाशय की विविधता को देखते हुए। हमने विकसित स्टोकेस्टिक का इस्तेमाल किया मॉडल मापदंडों से अनिश्चितता का प्रचार करने के लिए पीसी विस्तार पर आधारित तकनीक मॉडल भविष्यवाणियों के लिए। विकसित ढांचे का उपयोग वाणिज्यिक के साथ भी किया जा सकता है पेट्रोलियम जलाशयों के सिमुलेटर।

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## List of Acronyms

<b>IHTP</b> Inverse Heat Transfer Problems .....	4
<b>IHCP</b> Inverse Heat Conduction Problems .....	4
<b>BI</b> Bayesian inference .....	7
<b>MCMC</b> Markov chain Monte Carlo .....	9
<b>PCE</b> polynomial chaos expansions .....	10
<b>RML</b> Randomized maximum likelihood .....	17
<b>ENKF</b> Ensemble Kalman Filter .....	17
<b>ED</b> experimental design .....	18
<b>1-D</b> one-dimensional .....	56
<b>K-L</b> Karhunen-Loeve .....	56
<b>2-D</b> two-dimensional .....	56
<b>3-D</b> three-dimensional .....	56