

MAXIMALLY FLAT NONRECURSIVE DIGITAL FILTERS

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by
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July, 1985

DEDICATED

TO

MY PARENTS

CERTIFICATE

This is to certify that the thesis entitled, "Maximally Flat Nonrecursive Digital Filters" being submitted by B.C.Jinaga to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy is a record of bona fide research work carried out by him under my supervision and guidance and in my opinion, it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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ABSTRACT

Nonrecursive digital filters, characterized by finite duration impulse response, are unconditionally stable and can be designed for linear phase response. The various available design techniques, viz. windowing, frequency sampling and computer aided optimization, for finding suitable nonrecursive approximations to different idealized filter transfer functions, are not suitable for obtaining maximally flat response. Maximally flat filters are required in many situations because they give maximum attenuation in the stopbands and hence the best signal to noise ratio. For the recursive case, such filters can be derived directly from their analog counterparts by means of suitable transformations; this is not, however, possible for the nonrecursive case. A number of methods exist in the literature for designing maximally flat nonrecursive digital (MFNRD) filters; none of them, however, gives a direct and general method to determine the coefficients of these filters.

The research work presented in this thesis is concerned with the design of various kinds of MFNRD filters, viz. lowpass, highpass, bandpass and bandstop. Direct and general methods have been developed for determining the coefficients of these filters to meet the prescribed specifications. The development is in three stages. First, recursive relations and explicit formulas have been derived for the coefficients of MFNRD lowpass filters

of a given order, with specified degrees of flatness at $w = 0$ and $w = \pi$. Secondly, a method is developed for the design of MFNRD lowpass filters with specified cutoff frequency and transition bandwidth. It is then extended to the highpass case. Finally, using MFNRD highpass (lowpass) filters as prototypes, a technique is presented for the design of MFNRD bandpass (bandstop) filters with specified centre frequency, bandwidth and transition bandwidth. All of these methods, besides being simple and general, permit direct computation of the coefficients of the filters. A number of examples are given to illustrate the application of these methods. Schemes for implementation are presented and an error analysis has been carried out. A general FORTRAN program, catering to the design of MFNRD lowpass, highpass, bandpass and bandstop filters, with prescribed specifications, is given.

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