APPLICATION OF ANGULAR SPECTRUM TO OPTICAL COHERENCE AND NONLINEAR OPTICS

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PREFACE

The problem of the propagation of the electromagnetic waves in various media has been a subject of considerable investigation since the time of Maxwell. In such problems it is assumed that the field distribution is known over a given surface and it is required to obtain the field at any other point. The region of interest is supposed to be source free. Thus one is interested in solving the Helmholtz equation, satisfied by each of the frequency components of the electric field, subject to the following boundary conditions: (i) The wave field satisfies the Sommerfeld's radiation condition at infinity. (ii) The field reduces to the prespecified value at the given boundary.

Broadly speaking, there are two general methods for solving these boundary value problems. One of the methods is the Green's function method. The Green's function $G(\underline{r}, \underline{r}')$ represents the field at the observation point $\underline{r}$ due to a point source situated at $\underline{r}'$ and satisfies the boundary conditions mentioned above. The total field is obtained by integrating over the surface of the boundary. Though the method has the advantage of generality, the evaluation of Green's function is in general difficult. The other method is the method of separation of variables. Here the partial differential equation under consideration is broken up into a set of three ordinary differential equations. It is evident from the Helmholtz equation that there are two independent separation constants. The boundary conditions impose restrictions over the possible values of these separation constants. The allowed values are referred to as eigenvalues and the corresponding solutions are called eigenfunctions. Since the eigenfunctions form a complete set, the field distribution at any arbitrary space point can be expanded in terms of these eigenfunctions. The expansion coefficients are determined from the knowledge of the field at the given boundary. The evaluation of the expansion coefficients however, simplifies considerably when the boundary surface coincides with one of the coordinate surfaces.

Although in many problems of practical interest, the latter method is a simple one, it appears that in the treatment of the propagation problems only Green's function technique has been widely adopted. The present thesis is an attempt to demonstrate the applicability of the eigenfunction method instead of Green's function method for solving the propagation problems. In the simple case when the field is initially specified over a plane boundary and Cartesian coordinates are used as separation variables, it turns out that the eigenfunctions are just the plane waves propagating in different directions. The separation constants physically represent the direction cosines (angular dependence) of a particular plane wave component in a limited range of the eigenvalue spectra. Since the expansion coefficients in this case represent the contribution from different angular components, the expansion is usually referred to as the angular spectrum representation of the wavefield. Although we have briefly considered other types of boundaries and corresponding expansions also, in the present thesis we have mainly been concerned with the application of the angular spectrum to the propagation of electromagnetic waves in different media.

In the first chapter, we have briefly reviewed the angular spectrum representation. The concepts to be used in the following chapters are introduced. The representation is used to solve the Helmholtz equation and the solution is compared with the Rayleigh-Sommerfeld diffraction formula. It is shown that the two formulations are identical. The conditions for the validity of the angular spectrum are also discussed.

In the second chapter, the concept of the angular spectrum representation is employed to obtain the mutual coherence function in the far field diffraction plane. In analogy with the angular spectrum representation, an angular coherence function is introduced and some of its properties are considered. This angular coherence function is used to study the propagation of the mutual coherence function. The result is applied to several cases of particular interest such as the coherent, the incoherent and the homogeneous sources. We have also considered the case of a black body source with a circular aperture. It is found that even though the radiation is unpolarized at the aperture, it is partially polarized at some points in the far field diffraction plane.

In chapter 3, we extend the above treatment to study the propagation of the higher order coherence functions. The higher order coherence functions suitable for this purpose are introduced and some of their properties are considered. The general (n, m) th order coherence function is obtained in the observation plane in the far field approximation. The result is applied to several specific cases. We have used the general result to demonstrate that a thermal field remains thermal on propagation. The intensity fluctuations in the far field of a nonthermal source are also considered.

Chapter 4 deals with the propagation of electromagnetic waves in an inhomogeneous medium. Using the angular spectrum representation of the wavefield, we obtain an integro-differential equation satisfied by the angular amplitude for arbitrary inhomogeneities. This integro-differential equation is solved for two specific cases of interest namely that of a stratified medium and of a square low medium. In both cases an expression for the field distribution is obtained which expresses the field in terms of the boundary field through an integral. The kernel of the integral reduces to the free space propagation kernel in the limit when the inhomogeneities are absent. The result may be used to evaluate the coherence functions of various orders. In particular the average intensity in the far field is considered.

In chapter 5, angular spectrum representation is employed to study the effects of diffraction on the parametric processes in a nonlinear medium. These diffraction effects particularly become important in the far field region. We use the angular spectrum representation for the signal as well as the idler mode field. In order to simplify the treatment it is assumed that the pump mode field is still a plane wave and remains undepleted during interaction. The solutions are used to study the effect of diffraction on the conversion efficiency of a particular parametric process.

In chapter 6, we study the effect of pump depletion on the parametric processes from a quantum point of view. A trilinear Hamiltonian is employed and the three coupled Heisenberg equations of motion are solved in the short-time approximation. The solutions are used to obtain the time evolution of reduced density operators for the three interacting modes. This is done in terms of the diagonal coherent state representation of the density operator. It is assumed that the initial state of the system is a coherent state. The reduced density operator for the combined signal and the idler mode is also considered. It is found that the signal and the idler modes are uncoupled and behave independently.

Lastly in chapter 7, we consider the cylindrical wave expansion and the spherical wave expansion of the wavefields. It is shown that, as in the case of the plane wave expansion (angular spectrum representation) the cylindrical wave expansion also consists of two types of waves which we call the homogeneous cylindrical waves and the evanescent cylindrical waves.

The expansion is used to solve the Helmholtz equation when the field is initially specified over a cylindrical boundary. The result is illustrated with a few examples. In the last section, the spherical wave expansion is considered. It is found that in contrast to the plane wave expansion or the cylindrical wave expansion, no exponentially decaying waves exist in this case.

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