

# **SUB-SURFACE DRAINAGE OF TWO-LAYERED SOIL**

by  
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CERTIFICATE

This is to certify that the thesis entitled 'Sub-surface Drainage of Two-Layered Soil' being submitted by Harish Chandra Sharma, to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him under my guidance and supervision. In my opinion, the thesis has reached the requisite standard fulfilling the requirements of the regulation relating to the said degree. The material contained in this thesis has not been submitted, in part or full, to any other university or institute for the award of any degree or diploma.



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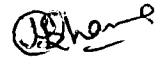
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SYNOPSIS

Application of irrigation water in excess of evapotranspiration demands is not unusual because of lack of perfect control over the water distribution and the difficulty of predicting the exact evapotranspiration needs. In due course of time, the soil profile may become nearly or completely saturated thus requiring removal of excess water by drainage for proper plant growth.

Drainage system is provided to ensure proper aeration in the root zone in humid regions and to prevent rise of salts in the soil profile in the arid regions. The sub-surface drainage is done either through open ditch drains or through tile drains. Although artificial drainage has been practiced for more than a century throughout the world (Luthin, 1959), research need for improving upon the drainage design still exists.

A soil strata requiring drainage is characterised by high water table. The purpose of sub-surface drainage is to lower excessively high water tables to a lower level within a pre-defined time period to prevent crop damage.

The design of sub-surface drainage system consists primarily of determining the proper drain depth and spacing to remove excess water. Different drainage parameters such as hydraulic conductivity, drainable porosity, depth to impermeable layer, specific discharge and mid-point depth of phreatic surface between drains, have to be known or determined to provide adequate drain spacing.

The theories available for the design of drainage system, have mostly been developed for homogeneous and isotropic soil. However, the homogeneous and isotropic medium is an ideal case which may not always be occurring in nature. A natural soil profile usually is composed of horizons or layers identified by distinct chemical and physical properties. The sedimentary deposits laid down in distinct layers have different hydraulic properties, thus resulting in layered soils. As a matter of fact most alluvial soils exhibit some degree of soil stratification. The occurrence of layered soils has been reported from different parts of the world.

In a layered soil, the permeability usually decreases downwards. A simple way of handling the layered soil problem for the design of sub-surface drainage has been to assume

the lower lesser permeable layer to be impermeable. This assumption has been applied in situations where the hydraulic conductivity of the lower layer is about one-fifth to one-tenth of that of the upper layer. It has been reported that this assumption leads to erroneous results.

A few investigators have presented analytical or experimental solutions for steady drainage of stratified soils. Kirkham (1951) analysed the problem of seepage into the tile drains in layered soils. Kirkham (1954) also reported the solution for the problem of simultaneous upward seepage of water from a ponded surface source into tube drains for a two-layered soil overlying the artesian gravel. Outmans (1964) generalized the discharge formulae for vertically and horizontally stratified soils in an earthen dam. Van Beers (1965) developed nomographs for computing drain spacing in stratified soils. Dagan (1965) developed analytical solution for the steady drainage of two-layered soils. The solution is the generalized ellipse equation for a two-layered soil. Toksoz and Kirkham (1971) applied potential theory for solving steady drainage problems of two and three layered soils. Najamii et al. (1978) presented potential theory solution for steady seepage to equally spaced horizontal tile drains in a two-layered soil replenished

by both upward and downward recharge. Walter et al. (1979) conducted field experiments to characterise the tile flow response in layered soils by taking 54 m x 61.5 m plots with the installation of 10 cm diameter tile-drains at the depth of 1 m. Khan (1983) experimentally verified the analytical solution of Toksoz and Kirkham (1971).

All the above mentioned solutions, for the sub-surface drainage of layered soils are applicable for the steady state condition only. Some of these solutions, based on potential theory, are complex in nature. No analytical solution exists for the transient sub-surface drainage conditions in layered soils, while the transient conditions are of common occurrence in humid areas.

Because of the above facts, the intended objectives for the present study were outlined as follows :

1. To prepare a state-of-the-art report on the drainage of layered soil.
2. To develop analytical solutions for the steady state sub-surface drainage, under constant rate of replenishment, for two-layered soil resting on a horizontal impermeable bed, for two cases :

when the water level in drain is higher than, and at the same level as the interface of the two layers.

3. To develop analytical solutions for the transient sub-surface drainage of two-layered soil, without replenishment for the two cases of water levels in the drains as mentioned at No. 2 above.
4. To use reservoir concept in evolving suitable model for the estimation of discharge in ditch drains in layered soil.
5. To present the developed relationships, for all the above cases, in non-dimensional form.
6. To estimate the extent of error involved in assuming the lower layer to be impermeable, when its hydraulic conductivity is one-fifth to one-tenth of the hydraulic conductivity of the upper layer.
7. To study the effect of variation in hydraulic conductivity ratios i.e.  $K_2/K_1$  on various drainage parameters.
8. To simulate two-layered soil medium in a vertical Hele-Shaw model and to carry out laboratory verification of the developed relationships for the

steady state and transient conditions of flow in sub-surface drainage at No. 2, 3 and 4, above.

The analytical solutions have been developed using Girinsiky potential (Girinsiky, 1946). The continuity equation, for one-dimensional steady ground water flow with replenishment, in terms of Girinsiky potential is given as :

$$\frac{\partial^2 G}{\partial x^2} + R = 0 \quad \dots (1)$$

where R is the uniform replenishment, and the Girinsiky potential, G, is an integral of various values of the potential at all the possible positions of the surface on the vertical line over  $0 < z < h$ . The Girinsiky potential for the phreatic surface in upper stratum of the two-layered soil, under study, may be defined as :

$$G = K_2 \int_0^{h_0} (h - z) \cdot dz + K_1 \int_{h_0}^h (h - z) \cdot dz \quad \dots (2)$$

Similarly, the continuity equation in terms of Girinsiky potential for transient conditions, without replenishment, is written as :

$$\frac{\partial^2 G}{\partial x^2} = \frac{1}{s} \cdot \frac{\partial G}{\partial t} \quad \dots (3)$$

where

$$s = \frac{2K_2 h_o + K_1 (H + H_m - 2h_o)}{2 \mu_1} \dots (4)$$

$K_1$  and  $K_2$  = hydraulic conductivities of upper and lower layers, respectively,

$h_o$  = thickness of the lower layer,

$H$  = water level in the drain above the impermeable barrier,

$H_m$  = height of water table at mid-drain spacing, and

$\mu_1$  = drainable porosity of the upper layer.

The solutions for Eqs. (1) and (3) have been proposed for the two cases :

i) when the water level in the drains is above the interface of the two-layered soil, i.e.  $H > h_o$ , and

ii) when the water level in the drains is at the interface of the two-layered soil, i.e.  $H = h_o$ .

Solutions for the above two cases have been developed to compute the height of the water table above the impermeable layer, maximum height of the water table at mid-drain spacing, and the drain spacing. In case of transient drainage the relationships for the drain discharge have also been developed for the above mentioned two cases. The non-dimensional form of all the solutions have also been given.

On the basis of the proposed solutions, the error in the estimation of parameters has been computed on the assumption of the lower layer to be impermeable in case its hydraulic conductivity is one-fifth to one-tenth of that of the upper layer. The error has been computed from the following relationship :

$$\text{Percent error in estimation} = \frac{\text{Parameter estimated for layered soil} - \text{Parameter estimated after neglecting second soil layer}}{\text{Parameter estimated for layered soil}} \times 100$$

Conceptual representation of a hydrologic system by a reservoir has been used by hydrologists to simulate flood routing, basin response in case of flood events, catchment behaviour in case of a continuous flow model and also in computing ground water flow hydrographs. Hellinga (1952), Kraijenhoff (1958), de Zeeuw and Hellinga (1958), de Zeeuw (1973) etc. applied conceptual reservoir approach to study drainage behaviour of homogeneous isotropic soil.

In the present study, the above reservoir concept has been used for the estimation of drain discharge in two-layered soil by assuming two soil layers as two different reservoirs connected in parallel so that both reservoirs

discharge into one common ditch drain. For this purpose the models of Glover-Dumm (in Dumm, 1954), and de Zeeuw (in Kraijenhoff, 1973) have been tried. With the help of these two models a set of four models has been developed by permutation and combination for two reservoirs as follows :

- i) Model No. 1 (Glo-Glo model) : In this model it has been assumed that flow from both the layers is represented by the Glover-Dumm model.
- ii) Model No. 2 (Zee-Zee model) : In this model it has been assumed that flow from both the layers is represented by the de Zeeuw model.
- iii) Model No. 3 (Glo-Zee model) : In this model it has been assumed that flow from the upper layer follows Glover-Dumm model and that from the lower layer follows de Zeeuw model.
- iv) Model No. 4 (Zee-Glo model) : In this model it has been assumed that flow from the top layer follows de Zeeuw model and that from the lower layer follows the Glover-Dumm model.

The comparative performance of these models has been studied. Also, this concept has been extended for use in multi-layered soil.

The significant conclusions of the study are :

1. On the basis of analytical solutions obtained by using Girinsky potential, solutions based on reservoir approach and the experimental investigations it may be concluded that :

The analytical solutions can be successfully used for the prediction of phreatic surface. These analytical models give discharge rates higher than those observed in Hele-Shaw model and closer to the discharge rates obtained by using Glo-Glo model which is based on reservoir approach. For the prediction of discharge, Zee-Glo model of reservoir approach was found most suitable as it gives minimum prediction error in comparison to other models based on reservoir approach, considering the experimentally observed values as the base.

2. If the hydraulic conductivity of the lower layer is one-fifth to one-tenth of that of the lower layer it had been generally treated as impervious. This assumption leads to erroneous results. The estimation error in the computation of any parameter, by using analytical solutions and neglecting the lower layer on the basis of above assumption, would be

higher for smaller values of drain spacing. A trend of similar nature has been observed in case of estimation of maximum height of water table at the mid-drain spacing.

The maximum height of water table plays a significant role in the drainage design. For example, for estimation of non-dimensional drain spacing,  $L^*$  ( $= L/h_0$ ) for  $K_2/K_1 = 0.1$  and  $M$  [ $= (H_m - h_0)/h_0$ ] equal to 0.01 the error is of the order of about 78 %, while for  $M = 1$  it is about 9 % .

The above comparison illustrates that assuming lower layer to be impervious, (even for the case when  $K_2/K_1 = 0.1$ ), may result in significant errors for the values of  $M < 2.0$ .

3. The analysis, of the solutions developed in this study, showed that the effect of the ratio of hydraulic conductivities of the two layers ( $K_2/K_1$ ) is more pronounced for its values less than 2.0 i.e. for the drainage system where the hydraulic conductivity of the lower layer is less than two times the hydraulic conductivity of the upper layer. Thus, the build-up of water table due to replenishment would occur at a faster rate for the values of  $K_2/K_1$ , less than 2. For the values of  $K_2/K_1$  greater than 2, the order of build-up decreases with the increase in the value of  $K_2/K_1$ . The rate of build-up of maximum height of water table, at

mid-drain spacing, increases with the increase in replenishment rate and decrease in the value of  $K_2/K_1$ .

4. The hydraulic properties of the lower soil layer have more effect on the water table build-up as the water level in the drain is nearer to the lower layer. As the value of  $H/h_0$  increases i.e. water level moves away from the interface in vertical direction, the effect of  $K_2/K_1$  decreases and it becomes more or less negligible for  $H/h_0 > 3$  at lower replenishment rate and for  $H/h_0 > 4$  for higher replenishment rates.

5. As the hydraulic conductivity of the lower layer increases the drain spacing also increases due to faster drainage rate and slow build-up of water table.

6. In case of reservoir approach it has been observed that the effect of  $j_1/j_2$  ( $j_1$  is the reservoir coefficient of the upper soil layer =  $\mu_1 L^2/\pi^2 K_1 D_1$ , and similarly  $j_2$  is the reservoir coefficient of the lower soil layer =  $\mu_1 L^2/\pi^2 K_2 D_2$ ,  $D_1$  and  $D_2$  are the saturated thickness of the upper and the lower layers) value is more pronounced when the maximum height of water table is closer to the interface. If the maximum height of the water table at the mid-drain spacing is farther from the interface, the effect of  $j_1/j_2$  is comparatively less.

7. Initially, the results of all the models based on reservoir approach show the trend of increase in discharge with the increase in  $j_1/j_2$  values. But with the lapse of time the peak of the plots (discharge versus  $j_1/j_2$ ) moves towards smaller  $j_1/j_2$  values and after a particular  $j_1/j_2$  value the discharge decreases up to a particular level and then it becomes almost constant i.e. no effect of further increase in  $j_1/j_2$  values.

8. The percent error in the prediction  $\left[ \frac{\text{Observed}^* - \text{Predicted}}{\text{Observed}^*} \right]$  x 100 was calculated for all the sets. For the case of steady state the prediction error was less than 6 %.

9. In case of the transient drainage behaviour of two-layered soils the results show that for smaller values of elapsed time the predicted elevation of simulated water table closer to drain was lesser than that observed in the experiment. As the elapsed time increases the predicted values of elevation of water table come closer to the observed values.

10. To evaluate the performance of various reservoirs models three sets of the conditions (i.e.  $Y_1 = 1.0, 1.5$  and  $2.0$ ) were simulated. The results showed that Zee-Glo

model i.e. when the upper layer follows de Zeeuw model and the lower layer follows Glover-Dumm model, the computed values were closer to the ones observed in the experiment.

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\* Experimentally observed in Hele-Shaw model.

TABLE OF CONTENTS

	Page
CERTIFICATE	... i
ACKNOWLEDGEMENT	... ii
SYNOPSIS	... iv
TABLE OF CONTENTS	... xix
LIST OF TABLES	... xxiii
LIST OF ILLUSTRATIONS	... xxiv
LIST OF SYMBOLS	... xxx
1. INTRODUCTION	... 1
1.1 General	... 1
1.2 Drainage of Stratified Soils	... 3
1.3 Objectives	... 6
1.4 Prologue	... 7
2. STEADY STATE SUB-SURFACE DRAINAGE OF LAYERED SOILS	... 10
2.1 Review of Literature	... 10
2.2 Problem Formulation	... 30
2.2.1 Differential equation	... 32
2.3 Analytical Solution	... 34
2.3.1 Case I : When water level in the drains is above the contact plane of the two layers	... 35
2.3.2 Case II: When water level in the drains is at the level of contact plane of the two layers	... 39

	Page
2.4 Performance Evaluation ...	40
2.4.1 Error in the estimation of $L^*$ and $q_s^*$ ...	41
2.4.2 General performance ...	45
3. TRANSIENT SUB-SURFACE DRAINAGE OF LAYERED SOILS ...	53
3.1 Problem Formulation ...	53
3.1.1 Differential equation ...	53
3.1.2 Initial and boundary con- ditions. ...	55
3.2 Theoretical Investigations ...	56
3.2.1 Case I : When water level in the drains is above the contact plane of the two layers ...	58
3.2.2 Case II : When water level in the drains is at the contact plane of the two layers ...	63
3.3 Performance Evaluation ...	67
3.3.1 Errors in the estimation of $L^*$ and $q_t^*$ ...	67
3.3.2 General performance ...	69
4. LINEAR RESERVOIR CONCEPT ...	85
4.1 General ...	86
4.2 Reservoir Theory Applied to Sub- Surface Drainage - A Review ...	89
4.2.1 The Edelman model ...	91
4.2.2 The Kraijenhoff model ...	94
4.2.3 The de Zeeuw model ...	99

	Page
4.3 Model Synthesis for Stratified Soils ...	100
4.3.1 Model No.1 [Glo-Glo model] ...	103
4.3.2 Model No.2 [Zee-Zee model] ...	106
4.3.3 Model No.3 [Glo-Zee model] ...	107
4.3.4 Model No.4[Zee-Glo model ] ...	108
4.4 Performance Evaluation ...	110
5. VISCOUS FLUID MODEL ...	121
5.1 The State of the Art ...	121
5.2 Analogous Equations ...	132
5.3 Computation of Scales ...	134
5.4 Design of Hele-Shaw model ...	138
5.4.1 Constructional features of the model ...	138
5.4.2 Viscous fluid ...	141
5.5 Experimental Procedure ...	143
5.5.1 Steady state drainage ...	143
5.5.2 Transient drainage ...	145
5.6 Discussion of Results ...	145
5.6.1 Steady state phreatic surface ...	145
5.6.2 Transient phreatic surface ...	150
5.6.3 Drain discharge ...	164
6. SUMMARY AND CONCLUSIONS ...	172
6.1 General ...	172
6.2 Steady State Drainage ...	173

6.2.1	Error in the estimation of $L^*$ and $q_s^*$	...	174
6.2.2	Effect of $K_2/K_1$ on phreatic surface.	...	175
6.2.3	Effect of $K_2/K_1$ on $q_s^*$ and $L^*$	...	176
6.3	Transient Drainage	...	176
6.3.1	Effect of $K_2/K_1$ on non-dimensional maximum height of water table $Y_m$ .	...	176
6.3.2	Effect of $K_2/K_1$ on $q_t^*$ and $L^*$	...	177
6.4	Linear Reservoir Concept	...	178
6.4.1	Effect of $j_1/j_2$ on discharge rate.	...	178
6.5	Experimental Investigations	...	179
6.5.1	Steady-state phreatic surface	...	179
6.5.2	Transient phreatic surface	...	180
6.5.3	Reservoir models	...	181
	REFERENCES	...	183
	APPENDIX	...	196