SOME STUDIES ON NONLINEAR AXISYMMETRIC DEFORMATION OF CYLINDRICALLY ORTHOTROPIC CIRCULAR PLATES AND SHALLOW SPHERICAL SHELLS ON ELASTIC FOUNDATIONS

By

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CERTIFICATE

This is to certify that the thesis entitled "SOME STUDIES ON NONLINEAR AXISYMMETRIC DEFORMATION OF CYLINDRICALLY ORTHOTROPIC CIRCULAR PLATES AND SHALLOW SPHERICAL SHELLS ON ELASTIC FOUNDATIONS" being submitted by Mr. Rakesh Kumar Jain, for the award of the degree of Doctor of Philosophy, to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him. Mr. Rakesh Kumar Jain fulfils all the requirements of the regulations laid down for the degree. The contents of this study have not been submitted to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

The theory of thin plates and shells on elastic subgrades occupies a prominent place in contemporary structural mechanics. In recent years the development of solid-propellant rocket motors, the increasing use of soft filaments in aerospace structures, operational activities of large transport aircrafts on runways, foundation of deep wells, large vessels, and the building activities in the cold regions have intensified the need for solutions of various problems of plates and shells made of composite materialscontinuously supported by elastic or viscoelastic media. The problems of response and stability of these structural elements, undergoing moderately large deformation, subjected to static and dynamic loads are very important in civil, mechanical and aeronautical engineering. Moreover, the necessity of analysing the nonlinear static and dynamic behaviour of orthotropic plates and shells continuously supported by elastic media arises not only from the point of view of the versatility in their applications in several disciplines but also from the point of view of the interest they stimulate as classical problems in mechanics and engineering.

The object of the present thesis is to analyse some geometrically nonlinear problems of static and dynamic axisymmetric behaviour of thin elastic cylindrically orthotropic circular plates and shallow spherical shells resting on elastic subgrades, undergoing moderately large deflections. Based on Von Kármán-

Marguerre strain-displacement relations, the governing differential equation for axisymmetric, moderately large deformation of an elastic cylindrically orthotropic shallow spherical shell continuously supported by elastic media are used. Spatial and temporal discretizations of governing partial differential equations of motion are carried out employing finite degree Chebyshev polynomials and implicit Houbolt time marching scheme. Clamped and simply supported immovable outer edge conditions have been assumed for both circular plates and spherical shells with symmetry conditions at the centre. In case of annular plates and shells, the inner edge is assumed to be free.

The linear and nonlinear mathematical representations for the interaction of the supporting elastic media have been considered. Employing the linear Winkler and Pasternak elastic foundations, the problems of static and dynamic deflection response of orthotropic circular plates and shallow spherical shells with and without hole have been investigated. The effect of linear elastic foundation parameters, namely, Winkler and Pasternak, orthotropic parameter, annular ratio, shell geometric parameter, foundation mass parameter and structure damping on the nonlinear response has been studied. The static and dynamic snap-through buckling analyses of orthotropic shallow spherical shells with and without hole and resting on Winkler and Pasternak foundations are carried out. An attempt has been made to study the influence of linear

elastic foundation parameters, orthotropy and annular ratio on the snap-through buckling loads. The results have been compared with the results available.

The static and dynamic response studies of circular plates and shallow spherical shells on nonlinear elastic foundations have been carried out. The cubic geometrical nonlinearity in elastic foundation has been considered for the mathematical representation of interaction behaviour. The influence of non-linear foundation stiffness parameter, orthotropic parameter, shell geometric parameter on the static and dynamic response of circular plates and shallow spherical shells without hole has been investigated for both immovable clamped and simply supported edge conditions. The effect of foundation mass parameter and structure damping on the transient response of these structural elements subjected to step and sinusoidal pulse loadings has been studied.

The following research publications have been prepared till now from the present thesis:

1. Nath, Y. and Jain, R.K., Nonlinear Dynamic Analysis of Orthotropic Annular Plates Resting on Elastic Foundations, Earthquake Engng. Struct. Dyn., 11, 785-796, 1983.
2. Nath,Y. and Jain, R.K., Non-Linear Dynamic Analysis of Shallow Spherical Shells on Elastic Foundations, Int. J. Mech. Sci., 25, 409-419, 1983.

3. Nath, Y. and Jain, R.K., Influence of Foundation Mass on the Nonlinear Damped Response of Orthotropic Shallow Spherical Shells, Int. J. Mech. Sci., Accepted.

4. Jain, R.K. and Nath, Y., Nonlinear Axisymmetric Static Analysis of Shallow Spherical Shells on Winkler-Pasternak Foundation, J. Enrg. Resources Tech., Trans. ASME, Communicated.

5. Nath, Y. and Jain, R.K., Nonlinear Studies of Orthotropic Shallow Spherical Caps on Elastic Foundation, Int. J. Nonlin. Mech., Communicated.

6. Nath., Y. and Jain, R.K., Nonlinear Transient Analysis of Orthotropic Annular Shallow Spherical Shells on Elastic Foundations, J. Engng. Mech. Div., Proc. ASCE, Communicated.

7. Jain, R.K. and Nath, Y., Effect of Foundation Nonlinearity on the Nonlinear Transient Response of Orthotropic Shallow Spherical Shells, J. Appl. Mech., Trans. ASME, Communicated.

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