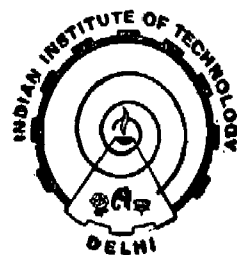


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CERTIFICATE

This is to certify that the thesis entitled
'Some Boundary Value Problems Associated With Dispersion
in Laminar Slip Flows', being submitted by Mr. Mahesh
Chandra Tiwari to the Indian Institute of Technology,
Delhi, for the award of the Ph.D. degree in mathematics,
is a record of bonafide research work carried out by him
under my supervision for the last three and a half year
and to the best of my knowledge it has reached the standard,
fulfilling the requirements of the regulations relating to
the degree.

I further certify that the results contained in
this thesis have not been submitted, either in part or in
full, to any other University or Institute for the award
of any Diploma or Degree.

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(MAHESH CHANDRA TIWARI)

ABSTRACT

The work presented in the thesis entitled 'Some Boundary Value Problems Associated With Dispersion in Laminar Slip Flows', is divided into five chapters. As a prelude to the problems investigated in this thesis, a comprehensive account of the related work in the field of channel flows including dispersion, chemical reaction and mass transfer across the porous bounding wall has been presented in the first introductory chapter of the thesis.

Chapter II, provides a systematic study of steady-state laminar slip flows of Newtonian fluid flowing in channels of different cross-sectional geometries and having one permeable bounding wall. The response of flow entities like fractional increase in flux due to velocity slip and friction factors ratio to variations in permeability and channel geometry parameters has also been discussed in detail in this chapter.

In Chapter III, the problems of steady and unsteady diffusion and convection in the presence of simultaneous homogeneous and heterogeneous chemical reactions in the liquid flowing through channels with one porous bounding wall are studied under laminar isothermal conditions. The steady-state problems have been treated analytically to obtain the solutions as eigen-function expansions containing confluent hypergeometric

functions. In the unsteady state case, the problems have been tackled by using Taylor's dispersion theory. The effect of velocity slip on concentration profiles and on effective Taylor diffusion coefficient has been investigated. The results obtained in this chapter generalize various well-known results.

Chapter IV deals with the study of laminar incompressible viscous flows through a porous medium of permeability k_s in channels having one porous bounding wall (permeability k). The velocity field solutions obtained in this chapter have been subsequently used to study the steady and unsteady dispersion of a soluble matter convectively diffusing in these flows. The steady-state problems, formulated as an eigen boundary value problems, have been solved numerically by finite-difference method. The concentration profiles have been analysed in detail, followed by a discussion of salient features arising due to the presence of parameters influencing concentration. The problems of unsteady dispersion of solute have been studied within the framework of Taylor's dispersion theory. The results obtained in this chapter generalize various known results in flow through channels of impermeable walls.

The last chapter of the thesis deals with the analysis of laminar flows of an incompressible viscous fluid through

porous media in closed and open parallel-plate channels with surface mass transfer which also takes into account the velocity slip at the surface of the porous bounding wall. The two-dimensional flows under study lead to fourth order nonlinear boundary value problems along with BJ slip boundary condition at the porous bounding wall and vanishing of stream-wise and transverse velocity components at the impermeable boundary. These problems have been solved numerically and flow velocity distributions predicted and analysed. The velocity field solutions have been employed to determine the concentration of soluble matter which convectively-diffuses in these flows.

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