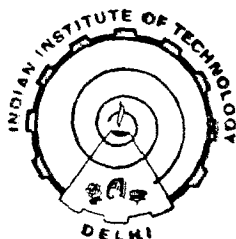


# ELASTO-PLASTIC FINITE ELEMENT ANALYSES OF SINGLE AND INTERACTING TUNNELS

by  
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A THESIS SUBMITTED TO  
THE INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
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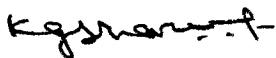
**Department of Civil Engineering**  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
January 1985

TO MY PARENTS

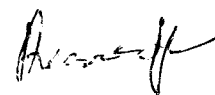
CERTIFICATE

This is to certify that the thesis entitled, "ELASTO-PLASTIC FINITE ELEMENT ANALYSES OF SINGLE AND INTERACTING TUNNELS" being submitted by Mr. R.K. Srivastava to the Indian Institute of Technology, Delhi, for the award of the degree of DOCTOR OF PHILOSOPHY is a record of the bonafide research work carried out by him. Mr. R.K. Srivastava has worked under our guidance for the submission of this thesis which to our knowledge has reached the requisite standard.

The thesis or any part thereof has not been submitted to any other university or institution for the award of any degree or diploma.



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## ABSTRACT

Hydroelectric projects are being planned, designed and constructed all over the world to augment the energy available from other sources. One of the important components of these projects is underground opening (tunnel). The underground openings may be required for installation of machineries or conveyance of water. In many hydroelectric projects multiple openings are encountered. A common problem is the requirement of understanding the interaction behaviour of adjacent openings.

Keeping the above facts in view, a systematic study has been planned to understand the behaviour of single and interacting deep underground openings (tunnels). Different aspects which affect the development of stresses and displacements around an opening viz., insitu stress condition, material behaviour, spacing between the interacting tunnels and sequence of excavation have been considered.

For the purpose of analysis, a computer program has been developed for the elasto-plastic finite element analysis of tunnels for plane strain condition. Process of excavation has been simulated in the finite element analysis. Elasto-viscoplasticity has been used as an artifice to obtain elasto-plastic solution.

A study has been carried out to compare Mohr-Coulomb, Drucker-Prager and Hoek-Brown yield criteria. From this, it has been concluded that the Hoek-Brown yield criterion is the most

suitable yield criterion to represent the strength behaviour of rocks. So, to represent the elasto-plastic behaviour of the geologic media, this yield criterion has been adopted for all the analyses.

For the analysis of single tunnels, two shapes, viz. circular and horse-shoe have been selected. In the case of interacting tunnels, circular tunnels have been chosen. In this case, the tunnels have been assumed to be excavated simultaneously and sequentially. Three pillar widths to tunnel diameter ( $W/D$ ) ratios of 0.3, 0.6 and 1.2 are considered. In each analysis, three insitu stress conditions viz., 0.5, 1.0 and 1.5 have been adopted. All the cases have been analysed considering the material behaviour as elastic as well as elasto-plastic.

In the case of elasto-plastic behaviour of the geologic media, the final displacements and stresses around the tunnel are found to be significantly affected by the sequence of excavation. The sequence of excavation results in development of tensile stresses and non-uniform displacements around the opening. Single stage excavation gives comparatively uniform displacements around the tunnel boundary. Among the two shapes of tunnels, horse-shoe shape is found to be less suitable for higher insitu stress ratio of 1.5.

In the case of interacting tunnels, it has been found that the interaction effect increases with decreasing  $W/D$  ratio for all insitu stress conditions. This interaction effect is higher for smaller insitu stress ratio ( $K_0 = 0.5$ ), if the material behaviour is elastic. In case the material behaviour is elasto-plastic, the

interaction effects are greater for the higher insitu stress ratio ( $K_0 = 1.5$ ). In general, the interaction effect is very much a function of insitu stress condition along with the space between the tunnels. The spacing required to reduce the interaction effect is higher for higher insitu stress ratios ( $K_0 = 1.5$ ). With increase in W/D ratio, the change from interacting to non-interacting (single) opening behaviour is not smooth.

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