NUMERICAL INVESTIGATION OF WAVE TURBULENCE IN NONLINEAR ACOUSTIC AND FLUID SYSTEMS

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by

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Submitted

in fulfillment of the requirements of the degree of

Doctor of Philosophy

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Dedicated to

my

beloved Family

Certificate

This is to certify that the thesis entitled **"Numerical Investigation of Wave Turbulence in Nonlinear Acoustic and Fluid Systems"** submitted by **Mr. Praveen Kumar** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy** in the Department of Energy Science and Engineering represents original research work carried out by him under my supervision and guidance.

To the best of my knowledge, the work presented in this thesis is genuine and has been completed in accordance with the academic requirements prescribed for the Ph.D. degree. The results and findings reported in this thesis have not been submitted, either wholly or partially, to any other university or institute for the award of any degree or diploma. I am satisfied with the quality and standard of the research work and recommend the thesis for the award of the aforesaid degree.

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Abstract

Wave turbulence represents a fundamental class of nonlinear phenomena in which energy is redistributed across a broad range of spatial and temporal scales through interacting dispersive waves. Such dynamics arise in diverse physical systems, including fluids, plasmas, nonlinear optical media, and Bose–Einstein condensates. Despite significant progress in weak wave turbulence theory, many realistic systems operate beyond idealized assumptions, where finite-amplitude effects, coherent structures, dissipation, and external driving play a crucial role in the transition from ordered wave motion to fully developed turbulence. This thesis presents a comprehensive numerical investigation of wave turbulence generation, stability, and energy spectrum evolution in fluid systems using nonlinear dispersive wave models derived from fundamental conservation laws.

The work begins with the derivation and numerical analysis of a modified nonlinear Schrödinger (MNLS) equation, obtained by incorporating energy conservation into the standard compressible fluid equations. In this formulation, the source of nonlinearity arises from temperature variations induced by high-amplitude acoustic waves. Numerical simulations reveal that, unlike the classical NLS equation—which exhibits Fermi–Pasta–Ulam (FPU) recurrence and quasi-periodic dynamics—the MNLS system undergoes a breakdown of recurrence due to enhanced nonlinearity, leading to the formation of localized structures and the onset of wave turbulence. A semi-analytical approach is employed to elucidate the physical mechanism governing the evolution, broadening, and steepening of localized wave packets. The resulting turbulence exhibits a power-law energy spectrum close to the Kolmogorov–Zakharov scaling, indicating a nonlinear cascade mediated by wave–wave interactions.

The thesis further extends the investigation to a Zakharov-like (ZL) model, derived from mass and momentum conservation equations, which captures the coupling between high-frequency acoustic waves and low-frequency density fluctuations. The model is shown to reduce to an MNLS equation under quasi-stationary assumptions. Linear stability analysis confirms the presence of modulational (Benjamin–Feir) instability, while numerical simulations demonstrate the formation of cavities, humps, and chaotic structures. The disruption of FPU recurrence and enhanced mode

coupling lead to broadband turbulence characterized by steeper energy spectra compared to the NLS case, reflecting stronger nonlinear interactions.

To examine the role of dissipation, a damped coupled wave–density model is developed and analyzed. The inclusion of damping significantly alters the turbulence dynamics by suppressing coherent recurrence, modifying instability growth rates, and regulating energy transfer across scales. Numerical results show that damping steepens the energy spectra and promotes statistically steady turbulence states consistent with weak wave turbulence predictions. The interplay between density and temperature fields is examined in detail, highlighting their combined influence on turbulence evolution and spectral behavior.

Finally, the thesis investigates a forced and damped Zakharov-type system, focusing on how external driving and frequency detuning control turbulence regimes. Three distinct regimes—cavity-dominated, mixed, and cascade-dominated turbulence—are identified based on detuning strength. Ensemble-averaged spectra reveal clear regime-dependent scaling laws, demonstrating how controlled forcing and dissipation shape nonlinear energy cascades.

Overall, this thesis establishes a unified numerical framework for understanding wave turbulence in fluid systems by systematically incorporating energy conservation, nonlinear coupling, damping, and driving into reduced wave models. The results provide new physical insights into the mechanisms governing instability, recurrence breakdown, and spectral energy transfer, bridging the gap between idealized wave turbulence theory and realistic fluid-wave dynamics. The findings are relevant to a wide range of applications, including acoustic turbulence, ocean and atmospheric waves, and other dispersive nonlinear systems.

सार

तरंग अशांति (Wave Turbulence) गैर–रेखीय भौतिक घटनाओं का एक मौलिक वर्ग है, जिसमें विविक्त तरंगों की पारस्परिक अंतःक्रिया के माध्यम से ऊर्जा का स्थानिक एवं कालिक पैमानों की विस्तृत श्रेणी में पुनर्वितरण होता है। इस प्रकार की गतिकी द्रव, प्लाज़्मा, गैर–रेखीय प्रकाशीय माध्यमों तथा बोस–आइंस्टीन संघननों जैसे अनेक भौतिक तंत्रों में पाई जाती है। यद्यपि दुर्बल तरंग अशांति सिद्धांत (Weak Wave Turbulence Theory) में उल्लेखनीय प्रगति हुई है, फिर भी अधिकांश यथार्थ भौतिक तंत्र आदर्श मान्यताओं से परे कार्य करते हैं, जहाँ सीमित आयाम प्रभाव, सुसंगत संरचनाएँ, अपव्यय तथा बाह्य प्रेरण तरंग गति से पूर्ण विकसित अशांति में संक्रमण को निर्णायक रूप से प्रभावित करते हैं। यह शोध–प्रबंध द्रव तंत्रों में तरंग अशांति की उत्पत्ति, स्थिरता तथा ऊर्जा स्पेक्ट्रम के विकास का एक व्यापक संख्यात्मक अध्ययन प्रस्तुत करता है, जो मूलभूत संरक्षण नियमों से व्युत्पन्न गैर–रेखीय विवर्तनशील तरंग मॉडलों पर आधारित है।

इस कार्य की शुरुआत एक संशोधित गैर–रेखीय श्रॉडिंगर (MNLS) समीकरण की व्युत्पत्ति और संख्यात्मक विश्लेषण से होती है, जिसे संपीड्य द्रव समीकरणों में ऊर्जा संरक्षण को सम्मिलित करके प्राप्त किया गया है। इस मॉडल में गैर–रेखीयता का स्रोत उच्च आयाम वाली ध्वनिक तरंगों द्वारा उत्पन्न तापमान परिवर्तन है। संख्यात्मक अनुकरणों से यह स्पष्ट होता है कि जहाँ पारंपरिक NLS समीकरण फर्मी–पास्ता–उलाम (FPU) पुनरावृत्ति और लगभग आवर्ती गतिकी प्रदर्शित करता है, वहीं MNLS तंत्र में प्रबल गैर–रेखीयता के कारण यह पुनरावृत्ति टूट जाती है, जिससे स्थानीयकृत संरचनाओं का निर्माण तथा तरंग अशांति का उद्भव होता है। एक अर्ध–विश्लेषणात्मक विधि का उपयोग स्थानीयकृत तरंग पैकेटों के प्रसार, संकेंद्रण तथा विस्तार की भौतिक प्रक्रिया को समझाने हेतु किया गया है। परिणामस्वरूप उत्पन्न अशांति का ऊर्जा स्पेक्ट्रम कोल्मोगोरोव–ज़खारोव स्केलिंग के समीप पाया गया, जो तरंग–तरंग अंतःक्रियाओं द्वारा नियंत्रित एक गैर–रेखीय ऊर्जा कैस्केड को दर्शाता है।

इसके पश्चात शोध को ज़खारोव–सदृश (Zakharov–like) मॉडल तक विस्तारित किया गया है, जिसे द्रव्यमान एवं संवेग संरक्षण समीकरणों से व्युत्पन्न किया गया है और जो उच्च–आवृत्ति ध्वनिक तरंगों तथा निम्न–आवृत्ति घनत्व उतार–चढ़ावों के युग्मन का वर्णन करता है। यह दर्शाया गया है कि अर्ध–स्थिर (quasi–stationary) अनुमानों के अंतर्गत यह मॉडल MNLS समीकरण में सन्निकट हो जाता है। रैखिक स्थिरता विश्लेषण से मॉड्यूलेशनल (बेंजामिन–फीयर) अस्थिरता की उपस्थिति की पुष्टि होती है, जबकि संख्यात्मक परिणाम गुहाओं, उभारों तथा अराजक संरचनाओं के निर्माण को प्रदर्शित करते हैं। FPU पुनरावृत्ति का विघटन और प्रबल मोड युग्मन व्यापक तरंग संख्या क्षेत्र में फैली अशांति उत्पन्न करता है, जिसका ऊर्जा स्पेक्ट्रम NLS तंत्र की तुलना में अधिक तीव्र पाया गया।

इसके अतिरिक्त, अपव्यय के प्रभावों का अध्ययन करने हेतु एक अपशोषित युग्मित तरंग–घनत्व मॉडल विकसित किया गया है। अपशोषण की उपस्थिति अशांति गतिकी को महत्वपूर्ण रूप से परिवर्तित करती है—यह सुसंगत पुनरावृत्तियों को दबाती है, अस्थिरता की वृद्धि दर को संशोधित करती है तथा पैमानों के बीच ऊर्जा स्थानांतरण को नियंत्रित करती है। संख्यात्मक परिणाम दर्शाते हैं कि अपशोषण ऊर्जा स्पेक्ट्रम को अधिक तीव्र बनाता है और दुर्बल तरंग अशांति सिद्धांत के अनुरूप सांख्यिकीय रूप से स्थिर अशांत अवस्थाओं को प्रोत्साहित करता है। घनत्व एवं तापमान क्षेत्रों की संयुक्त भूमिका का विस्तृत विश्लेषण किया गया है, जो अशांति के विकास और स्पेक्ट्रल व्यवहार को नियंत्रित करती है।

अंततः, शोध में एक प्रेरित एवं अपशोषित ज़खारोव–सदृश तंत्र का अध्ययन किया गया है, जिसमें यह विश्लेषण किया गया है कि बाह्य प्रेरण तथा आवृत्ति डिट्यूनिंग किस प्रकार विभिन्न अशांति अवस्थाओं को नियंत्रित करते हैं। डिट्यूनिंग की तीव्रता के आधार पर तीन भिन्न अशांति अवस्थाओं—गुहा–प्रधान, मिश्रित तथा कैस्केड–प्रधान—की पहचान की गई है। एन्सेम्बल–औसत ऊर्जा स्पेक्ट्रा स्पष्ट रूप से अवस्था–निर्भर स्केलिंग नियम प्रदर्शित करते हैं, जिससे यह स्पष्ट होता है कि नियंत्रित प्रेरण और अपव्यय किस प्रकार गैर–रेखीय ऊर्जा कैस्केड को आकार देते हैं।

समग्र रूप से, यह शोध–प्रबंध द्रव तंत्रों में तरंग अशांति को समझने हेतु एक एकीकृत संख्यात्मक ढाँचा प्रस्तुत करता है, जिसमें ऊर्जा संरक्षण, गैर–रेखीय युग्मन, अपशोषण तथा बाह्य प्रेरण को व्यवस्थित रूप से सम्मिलित किया गया है। प्राप्त परिणाम अस्थिरता, पुनरावृत्ति विघटन तथा स्पेक्ट्रल ऊर्जा स्थानांतरण की मूलभूत भौतिक प्रक्रियाओं पर नए दृष्टिकोण प्रदान करते हैं और आदर्श तरंग अशांति सिद्धांत तथा यथार्थ द्रव–तरंग गतिकी के बीच की खाई को पाटते हैं। ये निष्कर्ष ध्वनिक अशांति, महासागरीय एवं वायुमंडलीय तरंगों तथा अन्य विवर्तनशील गैर–रेखीय तंत्रों में व्यापक अनुप्रयोग रखते हैं।

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Abbreviations

AB	**A**khmediev **B**reather
BFI	**B**enjamin-**F**eir **I**nstability
CFD	**C**omputational **F**luid **D**ynamics
DNS	**D**irect **N**umerical **S**imulation
DZSE	**D**amped-**D**riven **Z**akharov **S**ystem of **E**quations
FD	**F**inite **D**ifference
FPU	**F**ermi-**P**asta-**U**lam
HF	**H**igh-**F**requency
KZ	**K**olmogorov-**Z**akharov
LF	**L**ow-**F**requency
MI	**M**odulational **I**nstability
MNLS	**M**odified **N**onlinear **S**chrodinger equation
mDZSE	**M**odified **D**amped-**D**riven **Z**akharov **S**ystem of **E**quations
NLS	**N**onlinear **S**chrodinger equation
OTSI	**O**scillating **T**wo-**S**tream **I**nstability
PBC	**P**eriodic **B**oundary **C**ondition
PDI	**P**arametric **D**ecay **I**nstability
ZL	**Z**akharov-**L**ike
ZSE	**Z**akharov **S**ystem of **E**quations

Physical Constants

Ambient sound speed used for acoustic normalization	c_0	$\sim$	10^4 cm s^{-1}
Boltzmann constant	k_B	$=$	1.380×10^{-16}erg K^{-1}
Equilibrium mass density used in simulations	ρ_0	$=$	1 g cm^{-3}
Reference angular frequency	ω_0	$\sim$	100 kHz
Reference spatial scale	x_n	$=$	0.07071 cm
Reference time scale	t_n	$=$	10 μs
Reference density scale	N_n	$=$	1 g cm^{-3}
Reference envelope scale	A_n	$=$	1 g cm^{-3}
Universal gas constant	R	$=$	8.314×10^7 erg mol^{-1} K^{-1}

Symbols

Symbol	Description	Typical use
$A(x,t)$	High-frequency acoustic-wave envelope or complex field amplitude	Zakharov models
A_0	Constant pump or forcing amplitude	driven systems
A_k	Fourier coefficient of the envelope field	spectral diagnostics
B_0	Initial wave amplitude	initial condition
c_0	Ambient or reference sound speed	acoustic scaling
c_1, c_2	Model coupling coefficients	reduced equations
$E(k), E_k$	Energy spectrum as a function of wavenumber	spectra
f	Width of localized wave structure	semi-analytical method
f_0	External forcing strength	damped-driven NLS
i or ι	Imaginary unit	complex fields
k	Wavenumber	Fourier space
k_x	Wavenumber in the x direction	one-dimensional spectra
k_0	Carrier wavenumber	envelope approximation
k_{MI}	Wavenumber associated with modulational instability	stability analysis
k_1	First daughter wavenumber in parametric decay	PDI analysis
K	Cascade or secondary interaction wavenumber	PDI/cascade

L	Length of the periodic computational domain	numerical setup
n	Low-frequency density perturbation or number of grid points, depending on context	model/numerics
N	Wave action or numerical grid resolution, depending on context	diagnostics
p	Pressure	fluid equations
T	Temperature field	thermodynamic coupling
$\tilde{T}$	Normalized temperature perturbation	coupled model
t	Time	physical/model time
u, v	Fluid velocity components or velocity field	hydrodynamics
x	Spatial coordinate	physical/model space
α	Initial perturbation wavenumber or fitted spectral exponent, depending on context	simulations/spectra
γ	Linear damping coefficient or growth rate, depending on context	damping/MI
γ_1, γ_2	Envelope and slow-mode damping rates	Zakharov-type models
γ_d, γ_e	Damping rates used in driven Zakharov simulations	numerical setup
δ	Damping coefficient in forced-damped NLS simulations	stability analysis
Δk	Spectral grid spacing	numerical setup
Δt	Time step	numerical setup
Δx	Spatial grid spacing	numerical setup
$\Delta\Omega$	Frequency detuning or mismatch between driver and carrier frequency	driven systems
ϵ	Small perturbation parameter or permittivity, depending on context	analysis
λ	Wavelength	wave description
$\rho(x,t)$	Mass density or density fluctuation field	fluid/Zakharov models

ρ_0	Equilibrium density	normalization
ρ'	Slowly varying density-envelope perturbation	derivation
σ	Sign of nonlinearity or complex modulation frequency, depending on context	NLS/MI
θ	Random phase variable in the initial condition	simulations
ω	Angular frequency	dispersion
ω_0	Carrier or driving angular frequency	envelope approximation
ω_a	Natural acoustic frequency	detuning
Ω_0	Nonlinear frequency shift	driven solutions
$\psi(x,t)$	Complex NLS envelope or normalized density field	NLS models
ψ_0	Background envelope amplitude	stability analysis
ψ_k	Fourier coefficient of ψ	spectra
ζ, τ	Normalized space and time variables	reduced NLS forms