

***-Prime and Strongly Prime Group Algebras and the Corresponding Radicals**

By

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Department of Mathematics

Submitted

*in fulfillment of the requirements
of the degree of Doctor of Philosophy*

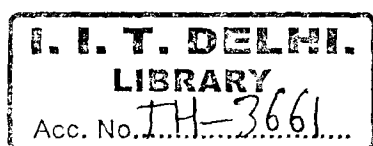
to the



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Group algebra

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To My Parents

CERTIFICATE

We are satisfied that the thesis entitled **-Prime and Strongly Prime Group Algebras and the Corresponding Radicals* presented by Ms. Kanchan Joshi (2003MAZ0010) is worthy of consideration for the award of the degree of Doctor of philosophy and is a record of the original bonafide research work carried out by her under our guidance and supervision and the results contained in it have not been submitted in part or full to any other university or Institute for award of any degree/diploma.

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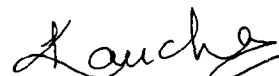
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ABSTRACT

In this thesis, we study $*$ -prime and strongly prime group algebras and the corresponding radicals. A ring R is said to be (right) strongly prime if for every $0 \neq r \in R$ there exists a finite subset X of R such that $rXt = 0, t \in R \implies t = 0$. Equivalently, R is (right) strongly prime if and only if for every non-zero ideal I of R , there exists a finite subset $X \subseteq I$ such that the right annihilator of X is zero. A ring R is said to be (right) $*$ -prime if for every $0 \neq r \in R$, there exists a finitely generated subring S of R such that $rSt = 0, t \in R \implies t = 0$. Strongly prime rings are $*$ -prime and $*$ -prime rings are prime. Strongly prime rings were first systematically studied by Handelman and Lawrence(1975). They proved that the group ring RG is strongly prime implies that R is strongly prime and the locally finite radical $L(G) = (1)$. Also they conjectured that the converse is also true. It is well known that RG is prime if and only if R is prime and $\Delta^+(G) = (1)$. Also group theoretically we have $\Delta^+(G) \subseteq \Lambda^+(G) \subseteq L(G)$. Here $\Lambda(G) = \{x \in G \mid x \text{ has finitely many conjugates under every finitely generated subgroup } H \text{ of } G\}$ and $\Lambda^+(G)$ is the torsion subgroup of $\Lambda(G)$. In view of above it is natural to study group algebras of groups G with $\Lambda^+(G) = (1)$.

We have given a complete characterization of $*$ -prime group rings. We prove that the group ring RG of the group G over the ring R is $*$ -prime if and only if R is $*$ -prime and $\Lambda^+(G) = (1)$. Using standard wreath product of groups, we have

constructed large number of groups for which the group algebra is $\ast$ -prime but not strongly prime. Also we have given certain classes of groups for which the group algebra is prime but not $\ast$ -prime. Intersection theorems play an important role in our study. We have obtained three new intersection theorems. Using our intersection theorems and some well known intersection theorems, we investigate the algebraic structure of $\ast$ -prime and strongly prime group algebras over fields. For a group G having a finite subnormal series with locally FC factors, we have established that KG is strongly prime if and only if $L(G) = (1)$.

Further, we have studied the strongly prime radical and the $\ast$ -prime radical of group algebras. Elementwise characterization of the $\ast$ -prime radical is given and its relation with some well known radicals is discussed. When S over R is a finite normalizing extension of rings, the relationship between the $\ast$ -prime radical of S and R is obtained. We have also investigated semi-strongly prime group algebras, that is, group algebras for which the strongly prime radical is zero. We would like to mention that the characterization of semiprime group algebras is already known. We have studied certain classes of groups for which the group algebra is semi-strongly prime if and only if it is semiprime. For residually finite and residually hyper- Δ groups we have obtained conditions under which the group algebra is semi-strongly prime.

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