

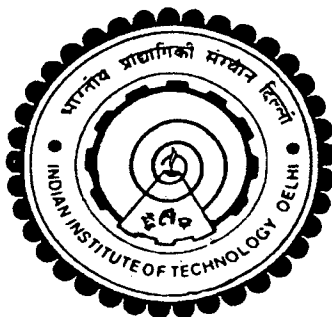
TOPOLOGY AND SHAPE OPTIMIZATION OF STRUCTURES

by

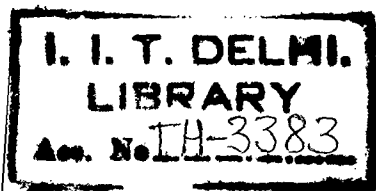
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DEPARTMENT OF APPLIED MECHANICS

submitted
in fulfillment of the requirements
for the degree of
DOCTOR OF PHILOSOPHY

to the



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FEBRUARY, 2006



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DEDICATED TO MY BELOVED PARENTS

FATHER SHRI. P. RAMASAMY

MOTHER SMT. R. RATHINAM

CERTIFICATE

This is to certify that the thesis entitled **Topology and Shape Optimization of Structures** being submitted by Mr. R. Balamurugan to the Indian Institute of Technology, Delhi for the award of Doctor of Philosophy in Applied Mechanics Department is a record of bonafide research work carried out by him. He has worked under my guidance and has fulfilled the requirements for the submission of thesis, which, in my opinion, has reached the requisite standard.

The results contained in this thesis have not been submitted in part or full, to any University or Institute for the award of degree or diploma.



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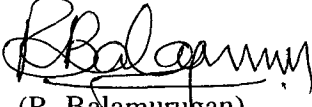
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(R. Balamurugan)

ABSTRACT

The complete structural design optimization process can be classified in to three categories namely topology optimization, size optimization and shape optimization. Topology optimization aims at finding the best topological connectivity for a structure to withstand the external load under specified boundary conditions. Size and shape optimizations are carried out on the fixed topology for finding the optimum cross sectional properties and boundary profiles. Topology optimization has been identified to be the most rewarding and also the most challenging task. Most of the methods available today for topology optimization have limited applicability and either converges to local optima, or, in the other extreme, do not explore other possible equally good designs.

Genetic Algorithm (GA) has been successfully used to overcome the above limitations and has been the active area of research since early nineties. The bit-array representation method in which the design domain is discretized into a finite number of cells and each cell is filled with void (0) or solid (1) has been widely used because the method is simple, robust and do not restrict the design space. The current state of the art reveals, the higher computational cost and noisy solutions that contain lot of cellular holes, as the main drawbacks of the GA based topology optimization method. The present investigation, aims at finding possible ways to overcome these drawbacks of the GA based topology optimization method, and presents a GA based Integrated Topology and Shape Optimization system.

It is known that the GA generally converges quickly in the initial stages but further progress towards the global optimal solution is very slow. In the present

work detailed studies on this characteristic of GA based bit-array represented topology optimization method have been carried out using digital image processing techniques and a two phase method has been proposed. The first phase uses GA based bit-array representation method while the second phase uses a GA based geometric representation method. The two phase method has been tested on various benchmark problems and always reaches global optimal solutions in an efficient manner than the various other bit-array based methods.

The use of Sequential Integer Programming (SIP) technique for the geometric representation based second phase of the two phase topology optimization method has been explored in detail based on which a Hybrid GA (HGA) based method has been proposed for topology optimization. The HGA method has drastically reduced the computational time needed for the geometric representation phase.

The use and suitability, of the various advanced parallel and adaptive algorithms that can be quite efficient and effective for improving the performance of the GA based topology optimization process, have been studied. A tailor made Two Stage Adaptive Genetic Algorithm (TSAGA) consisting Self Adaptive Island GA (SAIGA) and Adaptive GA (AGA) features has been proposed for the use in bit-array represented topology optimization method. The algorithm has been tested on a variety of 2D benchmark problems including the complex multiple loading case problem and the results are compared with other advanced GA algorithms. The two phase method along with TSAGA for the first phase offers global optimal solutions that are completely free of noise.

The problem of finding optimal boundary shapes of mechanical components and structures has been solved using GA without encountering the difficulties of design sensitivity calculations. Detailed studies on 2D benchmark problems have been carried out and practical profiles with smooth boundary shape have been obtained.

A GA based Integrated Topology and Shape Optimization system has been developed which consists of a Topology Optimization Block, a Shape Optimization Block and a Graphics and Image Processing Block. The optimal topology obtained from the topology optimization block is sent to Shape Design Block for subsequent shape optimization. The procedure has been tested on various single loading case problems with different volume fractions and multiple loading case problems. Practical designs with smooth boundary profiles good enough for manufacturing have been obtained.

The algorithms and computer programs needed for the application of the proposed two phase topology optimization method for solving 3D problems have been developed successfully and tested.

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