

EXPERIMENTAL AND FINITE ELEMENT ANALYSIS OF  
SKEW GIRDER BRIDGES

RAVINDRA SWAROOP

Department of Applied Mechanics

Submitted

in fulfilment of the requirements of  
the degree of Doctor of Philosophy

to the

Indian Institute of Technology, Delhi

New Delhi 110 016, India

April 1981

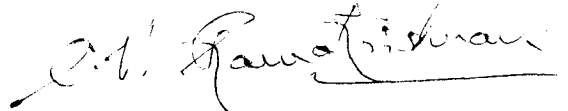
CERTIFICATE

This is to certify that the thesis entitled 'Experimental and Finite Element Analysis of Skew Girder Bridges' being submitted by Mr. Ravindra Swaroop to the Indian Institute of Technology, Delhi, India, for the award of the degree of Doctor of Philosophy in Applied Mechanics, is a record of bonafide research work carried out by him under our supervision and guidance. The thesis work in our opinion has reached the standard fulfilling the requirements for Doctor of Philosophy degree. The research report and the results presented in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.



Dr. C.S. Surana

Professor  
Dept. of Civil Engineering  
Indian Institute of Technology  
New Delhi 110 016, India



Dr. C.V. Ramakrishnan

Professor  
Dept. of Applied Mechanics  
Indian Institute of Technology  
New Delhi 110 016, India

## ACKNOWLEDGEMENTS

The author is very much indebted to Professor C.V. Ramakrishnan and Professor C.S. Surana for their personal supervision and guidance for this thesis work. Their valuable suggestions and encouragement were of immense help in planning and carrying out the project.

The author is thankful to Professor R.C. Malhotra, Professor M.C. Chaturvedi, Professor S.M.A. Kazimi, Professor R. Natarajan and Professor K.L. Kumar for taking personal interest in this project.

The author is also indebted to Professor A.K. Basu, Professor S.K. Gulhati and Professor Subhash Chander, for their kind permission for carrying out the experimental work in the Structures Laboratory of the Institute. The valuable suggestions of Professor B.M. Ahuja, in carrying out the experimental investigations are gratefully acknowledged.

During the experimental work, the cooperation and help extended by the staff of the Structures Laboratory and Workshop of Civil Engineering Department is gratefully acknowledged. The author is also thankful to the staff of the Photographic Section of the Institute for doing the photographic work.

The author takes this opportunity to thank the Director of the Institute. The help rendered by the staff of the Computer Centre of the Institute is gratefully acknowledged.

The author is very grateful to the management of M/s. National Buildings Construction Corporation Ltd., New Delhi, especially Mr. P.N. Sadhu, Mr. B.P. Dave and Mr. R.K. Gupta, for granting the necessary leave.

Thanks are also due to Mr. P.M.P. Nambiar for meticulously typing this thesis and Mr. B.B. Arora for tracing the drawings.

The author thanks his wife, family members and relatives for their constant encouragement and help for successful completion of this thesis work.

RAVINDRA SWAROOP

## ABSTRACT

This thesis describes the theoretical and experimental analysis of skew girder bridges. For the purpose of theoretical analysis a finite element based computer program has been developed. The accuracy of the program has been tested using several standard examples. Automated mesh generation facility has been provided to handle the analysis of three and four girder skew bridges. The program is suitably modified to handle a group of moving patch loads. Such a computer program is useful from the point of view of design.

Studies for girder bending moments have been carried out on three and four girder bridges by applying firstly a concentrated load at midspan of an exterior girder, secondly by applying a concentrated load at midspan of an interior girder and thirdly by analysing the bridges for their dead loads. The studies are carried out by varying one parameter at a time and applying one load at a time. The parameters varied are : skew angle, girder spacing to span ratio, ratio of flexural rigidity of transverse medium to flexural rigidity of longitudinal girder. The variations of girder bending moments have been plotted along the span of each girder of each bridge analysed. These studies are useful for understanding the distribution of bending moments among the girders of skew bridges.

Skew girder bridges having transversals perpendicular to the girders and support diaphragms parallel to the abutments have three

types of slab panels viz., triangular, trapezoidal and rectangular. Parametric studies for slab bending moments have been conducted on three girder bridges. In each study the panel under consideration is first analysed by applying a concentrated load at the centroid of the panel and then by applying the same load as pressure load on the entire panel. This type of loading is chosen because the behaviour of the panels under patch load is expected to lie between that under concentrated load and pressure load. These studies are useful to identify the critical slab panels for different combinations of bridge parameters.

In an actual bridge the deck slab is subjected to patch loads. To study the effect of the size of patch, load is applied through three different sizes of patch on the centre of a rectangular panel of slab of a  $45^{\circ}$ -skew girder bridge. The finite element results for patch loads are compared with those obtained using Pigeaud's curves. Discrepancies have been observed between the results of the two methods.

Experimental investigations have been carried out on a R.C. skew bridge model. The model has three girders, three transversals and two support diaphragms. The scale ratio of the model is  $1/6$ . For testing the bridge, suitable loading and supporting frames of structural steel sections have been designed and fabricated. The bridge has been tested for six positions of load on the girders and two positions of load on the slab. The experimental results of the

various influences measured have been compared with those obtained using the computer program and generally a good agreement has been observed.

To study slab strains in a greater detail, experiments have also been conducted on an available Sand Araldite model of a  $45^{\circ}$ -skew girder bridge having three girders. Patch load is applied directly on the slab of this bridge. The slab strains are measured directly under the patch load and compared with those obtained using the computer program. For concentrated loads applied on the girders, other influences like girder soffit strains and support reactions have also been measured and compared with those obtained using the computer program for this Sand Araldite model. The agreement between the experimental and theoretical results has been very good.

CONTENTS

	Page
ACKNOWLEDGEMENTS	(ii)
ABSTRACT	(iv)
LIST OF TABLES (Tables are positioned at the end of each Chapter)	(xiii)
LIST OF FIGURES AND PHOTOGRAPHS (Figures and Photographs are positioned at the end of each Chapter)	(xvi)
LIST OF PRINCIPAL SYMBOLS	(xxvii)
CHAPTER 1	
INTRODUCTION AND LITERATURE SURVEY	1
1.1 INTRODUCTION	1
1.2 HISTORICAL REVIEW	4
1.2.1 Method of Harmonic Analysis	4
1.2.2 Finite Difference Method	6
1.2.3 Grillage Analysis	6
1.2.4 General Shortcomings	9
1.2.5 Finite Element Method	9
1.2.6 Review of Experimental Work	11
1.3 OBJECT AND SCOPE	18
1.4 OUTLINE OF THE THESIS	19

	Page
CHAPTER 2	
THEORY OF FINITE ELEMENT METHOD	22
2.1 INTRODUCTION	22
2.2 STRUCTURAL IDEALISATION	22
2.3 BASIC THEORY OF THE FINITE ELEMENT METHOD	23
2.4 REVIEW OF PLATE BENDING ELEMENTS	27
CHAPTER 3	
FINITE ELEMENT COMPUTER PROGRAM FOR SKEW GIRDER BRIDGES	32
3.1 INTRODUCTION	32
3.2 ECCENTRICALLY STIFFENED SLAB BEAM SYSTEM	32
3.3 THE MIXOPARAMETRIC QUADRILATERAL PLATE BENDING ELEMENT	34
3.3.1 Shape Functions for Four Noded Isoparametric Quadrilateral Element	37
3.3.2 Cartesian Derivatives of G and H	37
3.3.3 Derivation of [ C ] Matrix	39
3.3.4 Derivation of [ Q ] Matrix	40
3.4 LINEAR ISOPARAMETRIC QUADRILATERAL PLANE STRESS ELEMENT	41
3.5 ECCENTRIC BEAM ELEMENT	43
3.5.1 Derivation of Displacement and Force Transformation Matrices	46
3.5.2 Derivation of Global Transformation Matrix	49
3.5.3 Derivation of Global Stiffness Matrix of Eccentric Beam Element	50

	Page
3.6 COMPUTER PROGRAM FOR LINEAR ELASTIC ANALYSIS OF SKEW GIRDER BRIDGES	51
3.7 CONVERGENCE STUDIES	58
3.7.1 Square Plate	58
3.7.2 T-Beam	58
3.7.3 45 <sup>0</sup> -R.C. Skew Girder Bridge Model	59
3.8 CONCLUSIONS	60
 CHAPTER 4	
DEVELOPMENT OF COMPUTER PROGRAM FOR HANDLING PATCH LOADS AND GROUP OF MOVING PATCH LOADS	94
4.1 INTRODUCTION	94
4.2 CALCULATION OF EQUIVALENT NODAL LOADS	95
4.3 COMPUTER PROGRAM	96
4.4 TEST PROBLEMS	100
4.5 CONCLUSIONS	103
 CHAPTER 5	
PARAMETRIC STUDIES FOR GIRDER MOMENTS	119
5.1 INTRODUCTION	119
5.2 VARIATION OF GIRDER BENDING MOMENTS WITH CHANGE IN SKEW ANGLE	120
5.2.1 Three Girder Bridge	121
5.2.2 Four Girder Bridge	121

	Page
5.3 VARIATION OF GIRDER BENDING MOMENTS WITH CHANGE IN GIRDER SPACING TO SPAN RATIO	122
5.3.1 Three Girder Bridge	123
5.3.2 Four Girder Bridge	124
5.4 VARIATION OF GIRDER BENDING MOMENTS WITH CHANGE IN RATIO OF TOTAL FLEXURAL RIGIDITY OF TRANSVERSE MEDIUM TO FLEXURAL RIGIDITY OF A GIRDER	124
5.4.1 Three Girder Bridge	125
5.4.2 Four Girder Bridge	126
5.5 DISCUSSIONS AND CONCLUSIONS	127
 CHAPTER 6	
PARAMETRIC STUDIES FOR SLAB MOMENTS IN GIRDER BRIDGES	156
6.1 INTRODUCTION	156
6.2 VARIATION OF SLAB MOMENTS WITH CHANGE IN RATIO OF TOTAL FLEXURAL RIGIDITY OF TRANSVERSE MEDIUM TO FLEXURAL RIGIDITY OF A GIRDER	157
6.3 VARIATION OF SLAB MOMENTS WITH CHANGE IN SKEW ANGLE	159
6.4 VARIATION OF SLAB MOMENTS WITH CHANGE IN THE DIMENSIONS OF TYRE CONTACT AREA	159
6.5 DISCUSSIONS AND CONCLUSIONS	161
 CHAPTER 7	
EXPERIMENTAL STUDY	182
7.1 INTRODUCTION	182
7.2 DESCRIPTION OF R.C. BRIDGE MODEL	182

	Page
7.3 DESIGN AND CONSTRUCTION OF LOADING FRAME	184
7.4 DESIGN AND CONSTRUCTION OF SUPPORTING FRAME	187
7.5 MEASURING INSTRUMENTS AND INSTRUMENTATION	188
7.6 CONTROL TESTS	196
7.6.1 Modulus of Elasticity of Steel	196
7.6.2 Compressive Strength of Concrete	197
7.6.3 Modulus of Elasticity of Concrete	197
7.6.4 Modulus of Rupture of Concrete	198
7.7 TESTS ON THE MODEL	198
7.7.1 Cracking Test	198
7.7.2 Single Point Load Tests	200
7.7.3 Patch Load Tests on the Slab	201
7.8 TESTS ON A SAND ARALDITE SKEW BRIDGE MODEL	202
7.9 SUMMARY AND CONCLUSIONS	205
 CHAPTER 8	
COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS	220
8.1 INTRODUCTION	220
8.2 R.C. BRIDGE MODEL	220
8.2.1 Deflections of Girders	221
8.2.2 Rotations of Girders	225
8.2.3 Girder Strains at Bottom Reinforcement Level	229

	Page
8.2.4 Slab Strains due to Patch Load	232
8.2.5 Support Reactions .	234
8.3 SAND ARALDITE BRIDGE MODEL	235
8.3.1 Deflections of Girders	235
8.3.2 Support Reactions due to Load on Girders	235
8.3.3 Slab Strains and Girder Soffit Strains due to Load on Girders	236
8.3.4 Slab Strains due to Patch Load on the Slab	237
8.4 DISCUSSIONS AND CONCLUSIONS	238
8.4.1 R.C. Bridge Model	238
8.4.2 Sand Araldite Bridge Model	240
CHAPTER 9	
SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH	285
9.1 SUMMARY AND CONCLUSIONS	285
9.2 CONTRIBUTIONS OF THE PRESENT STUDY	290
9.3 SUGGESTIONS FOR FUTURE RESEARCH	291
APPENDIX I	
CALCULATION OF PARTIAL DERIVATIVES OF $[J]^{-1}$ W.R.T. G AND H	293
APPENDIX II	
DETAILED CALCULATIONS FOR $[Q]$ MATRIX	295
REFERENCES	298
VITA	307
PAPERS BASED ON PRESENT INVESTIGATIONS	308