ANT TRAIL FORMATION: ANALYSIS, ALGORITHMS AND APPLICATIONS

by

SAMEENA SHAH

DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfilment of the requirements of the degree of

Doctor of Philosophy

to the

Indian Institute Of Technology Delhi

Hauz Khas, New Delhi-110016, India

January 2011

Dedicated to
everyone who has ever inspired me

Certificate

This is to certify that the thesis entitled **"Ant Trail Formation: Analysis, Algorithms and Applications"**, which is being submitted by **Sameena Shah** for the award of the degree of **Doctor of Philosophy in Electrical Engineering** to the **Indian Institute of Technology Delhi**, is a bona fide research work done under our guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other Institute for the award of any degree or diploma.

(Jayadeva)
Department of Electrical Engineering
Indian Institute of Technology Delhi
Hauz Khas, New Delhi-110016
India

(Ravi Kothari)
IBM India Research Lab
4, Block C, Institutional Area
Vasant Kunj, New Delhi-110070
India

(Suresh Chandra)
Department of Mathematics
Indian Institute of Technology Delhi
Hauz Khas, New Delhi-110016
India

Acknowledgements

PhD is a long journey; a journey that I feel blessed to have enjoyed. My thesis is about Ant algorithms, but that is only a fraction of what I have learnt in this process. It is strange to write this acknowledgement, it tells me that it is the end of an era that had absorbed me for all these years. That is a strange feeling indeed!

Of course, I was not alone in this, there were those who were guiding me throughout. It would be an understatement to say that this dissertation would not have been possible in its present form had it not been for the guidance and motivation from my supervisors. I am very lucky to have three wonderful supervisors. Prof. Jayadeva, Dr. Ravi Kothari, and Prof. Suresh Chandra are incredible people whom I still have a lot to learn from. Their belief in me made me achieve more than I could have. Their motivation and passion rubbed off on me, and made me strive for more.

I thank all my Student Research Committee members, Prof. R.K. Patney, Prof. R.K.P Bhatt and Dr. Aparna Mehra, for taking out time to listen to my ideas. They provided many valuable suggestions and corrected me whenever I went astray. I also thank Prof. G.S. Visweswaran, Prof. S.D. Joshi, and Prof. B. Bhaumik for their encouragement. I also thank the Dept. of Electrical Engineering and all the technical staff for their help on numerous occasions.

I had some wonderful friends to share this experience. My fondest memories include tea break discussions with Dr. Lalit Jiwani, Dr. Alok Kanti Deb, Dr. Sukhendu Deb Roy, Poornima Agarwal, and Dr. Jency Thomas. My lab mates Preeti Singh, Radhakrishnan Sithanandam, Sadashiva Chakrasali, Nitin Goyal and Kanika Nadda brought vibrant energy and enthusiasm to the lab.

It is difficult to come up with words to thank one's family. Their constant support, unconditional love, sacrifice, and so much more. But, in this platform, I thank them, the best parents one could possibly have, for instilling the value of education and hard work in me. My brothers Sameer and Jassim have always been my support, my strength and my reality check. I thank my sister-in-law Zeenat for being my elder sister and my friend. I thank my wonderful hus-

band Tanveer, who has had to assume several roles ranging from agony aunt to superman. With him by my side, I do not feel overwhelmed by anything.

Finally, I thank God for giving us this life, and its many unanswered questions.

Sameena Shah

Abstract

Nature is replete with examples where individual organisms perform distributed simple actions, that cooperatively lead to complex emergent phenomena. The flocking of birds, sorting of broods in colonies, herding, bacterial growth, and fish schooling are examples of such emergent phenomena. For example, to forage food, ants form a trail on the shortest known path. This optimization is achieved by using only indirect communication and local information, without explicit knowledge of the path lengths, and in the absence of any centralized command and control. Understanding the trail formation behaviour of ants is interesting as it may allow us to address other complex optimization and distributed control problems.

What is also interesting is that ants are not always able to form a trail on the shortest path known to the colony. This is because of an initial preference or 'bias' that develops in favour of longer paths discovered earlier. This initial bias prevents ant inspired algorithms from reaching the optimal solution. In this thesis, the first problem we address is the effect of the initial bias on the probability of trail formation on the optimal path. We relate the ant trail formation process to the Generalized Polya Urn problem, and derive explicit analytical results for the trail formation process. Specifically, we derive the pheromone distribution on equal length paths at any later time, and using the derived distribution, we derive the probability of convergence to a path.

We propose a method to incorporate the effect of path lengths on the trail formation process. Given the initial biases and lengths of available paths, we derive the resulting pheromone distribution at any later time. This probability is used to quantize the amount of bias on long paths, that can be reverted on account of the shortness of the optimal path. We show that beyond a threshold bias on long paths, the shortness of the optimal path alone will not suffice to bring the colony out of convergence to a suboptimal path.

We incorporate the effect of evaporation, and demonstrate its effect on the probability of convergence to the optimal path. We show that a mechanism such

as evaporation that removes pheromone from paths, can increase the probability of convergence to the optimal path, but this increase may not be enough to bring the system out of premature convergence to a sub-optimal path.

We propose a novel biologically inspired pheromone update rule that guarantees convergence to the optimal path. We prove that this update rule will cause the pheromone distribution to favour the shortest path irrespective of initial biases, problem structure, or explicit knowledge of good solutions. We also prove that this equilibrium is not a static one; if a shorter path is discovered, the entire colony will shift to it. We show that a family of variant rules also have similar properties.

Most optimization problems do not map to independent paths; rather they map to a graph topology with non-disjoint paths where edge weights correspond to solution costs. In a general graph based problem, overlapping edges may be present between paths. This results in non-uniform pheromone concentration across its length. We give the equivalent update rule for general graph based problems, and prove that asymptotically, the pheromone distribution will concentrate on the shortest path amongst the set of paths in a graph.

In many applications, it is desirable to choose several short paths instead of a single shortest path. For instance, to avoid congestion in network routing applications, to forage from multiple food sources, and to introduce redundancy in a system. We propose a novel ant pheromone update rule that asymptotically aligns the pheromone distribution with M shortest paths instead of a single shortest path.

Motivated by our novel pheromone rule, we propose a dynamic routing algorithm and assess its performance under different load and network conditions. Finally, we conclude with some observations and suggestions for future work.

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