

**MULTISCALE HYDROLOGICAL FORECASTING USING  
COUPLED WAVELET VOLTERRA MODELS**

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COUPLED WAVELET VOLTERRA MODELS**

By

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in fulfilment of the requirements of the degree of

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**To my beloved Teacher**

## **CERTIFICATE**

This is to certify that the thesis entitled '**Multiscale Hydrological Forecasting using Coupled Wavelet Volterra Model**' being submitted by **R. Maheswaran** to **Indian Institute of Technology Delhi, New Delhi (India)** for award of the degree of Doctor of Philosophy in Civil Engineering is a bonafide research work carried out by him under my supervision and guidance. The research reports and the results presented in this thesis have not been submitted, in parts or in full, to any other University or Institute for the award of any other degree or diploma.

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## ABSTRACT

Forecasting is critical in most planning studies and management tasks and so too in hydrological studies where forecasting plays a vital role in a variety of areas viz. flood and drought management, development of early warning systems, sedimentation rate forecasting and other reservoir studies, groundwater trends and climate change impact studies amongst others. However, the complexity of the natural (hydrological and geophysical) process and its variability in time and space makes the task of forecasting very challenging. The natural processes are generally composed of multiple features that are present at different temporal scales but whose individual, scale specific, characteristic details get camouflaged in the integrated observations. Generally, traditional models are seen to lack the capability to recognize and discriminate between features that operate at different scales and are therefore unable to provide satisfactory forecasts. The main aim of this study is to develop a Wavelets based, unified modelling framework that is able to detect the underlying, but individually non-observable, scale specific features in the hope to obtain more efficient forecasts.

In addition to its time localization capabilities, Wavelets are capable of yielding multi-resolution process decompositions and these advantages have encouraged the development of hybrid models (see, for example, Kim (2003), Kisi (2008), Partal and Kisi (2007), and Adamowski (2010)). The techniques and methodologies, developed in these latter studies, are based on a coupled Wavelets-ANN modelling framework and have been used for various forecasting applications. However, these models have often been criticized for various reasons and, in no small measure, on account of the (i) implied black box approach, (ii) heavy computational burden, (iii) excessive use of heuristics, and (iv) inability to operate in real time mode.

The present research is a result of the perceived need for a versatile forecasting framework that would have the capacity to accommodate these aforementioned concerns and has led to the present concept of a nonlinear adaptive Coupled Wavelet-Volterra (CWV and CWV-KF) forecasting model and the proposed methodology has been applied to four case studies in this thesis. The first two case studies deals with (i) monthly stream flow forecasting, and (ii) ground water levels forecasting. The third and the fourth case studies deal with multivariate monthly and daily stream flow forecasting respectively. Apart from these case studies, two additional time series having distinct non-stationary and nonlinear characteristics have also been analysed and discussed with results. The proposed model has also been compared with other models namely, Wavelet Linear Regression (WLR) models, ANN models, ARIMA models, Dynamic Auto Regressive (DAR) models, Wavelet-Neural Network (WA-ANN) models and the Dynamic Harmonic Regression (DHR) models and some of the salient results include:

- (1) In comparison with the WA-ANN models, the proposed CWV-KF model performed better and produced more accurate forecasts. For example, in Case Study I for Station I, the CWV-KF model results yielded a Nash-Sutcliffe Criterion (NSC) value of 0.86 compared to a value of 0.77 obtained using WA-ANN models.
- (2) Similarly in Case Study II, forecasting the ground water level series, the best CWV-KF model for both Wells I and II, had testing period NSC values of 0.95 and 0.84 respectively, and the proposed CWV-KF model is clearly seen to be superior than the best WA-ANN and WLR models which respectively reported NSC of 0.90 and 0.89 for Well I and an NSC values of 0.77 and 0.76 for Well II respectively.
- (3) The CWV-KF performed very well even for lead times up to 18 months.
- (4) In Case Study IV, the proposed model produced good results up to a lead time of 5 days when compared to the other models.

- (5) The proposed CWV model seems to need fewer parameters in comparison with WA-ANN models.
- (6) Further, this research addresses some of the fundamental issues that are generally ignored when using wavelets for forecasting applications. These issues relate to (i) selection of the mother wavelet function, (ii) boundary treatment, (iii) shift variance issues with wavelets, (iv) non-redundant decomposition related issues, and (v) number of relevant decomposition levels. The following are some of the salient results achieved:
- a) The redundant *à trous* wavelet transform is the most appropriate decomposition algorithm for forecasting purposes.
  - b) The depth of time series decomposition must be chosen after an investigation for the underlying dominant features in order to enable an unambiguous detection of these features.
  - c) The selection of mother wavelet plays an important role in forecasting and the following is concluded from the study:
    - (i) For time series having short lived and transient features, wavelets with compact support and reduced vanishing moments such as the Haar wavelet (db1) is recommended.
    - (ii) Wavelets with wider support and higher vanishing moments such as the db2 and spline-b3 wavelets are recommended for time series having long term memory and nonlinear features as seen from the obtained prediction accuracy.

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