

A THESIS ON  
NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

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C E R T I F I C A T E

This is to certify that the thesis entitled 'Numerical Solution of Differential Equations' which is being submitted by Mr. Krishan Dev Sharma for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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## A C K N O W L E D G E M E N T S

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