

ANALYTICAL AND NUMERICAL STUDIES OF SOME NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS IN FLUID FLOWS

By

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*Thesis submitted
in fulfilment of the requirements
for the award of the Degree of
DOCTOR OF PHILOSOPHY*



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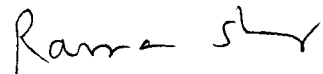
AUGUST, 1995

*Dedicated to
My Maa and Chaachi*

CERTIFICATE

This is to certify that the thesis entitled "ANALYTICAL AND NUMERICAL STUDIES OF SOME NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS IN FLUID FLOWS" which is being submitted by Mr. Dheeraj Bhardwaj for the award of DOCTOR OF PHILOSOPHY (MATHEMATICS) to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him under my guidance and supervision.

The thesis has reached the standard, fulfilling the requirements of the regulating relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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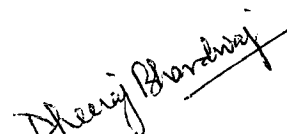
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