NUMERICAL MODELLING OF OXYGEN AND CARBON DIOXIDE TRANSPORT IN THE PULMONARY CIRCULATION

by

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CERTIFICATE

This is to certify that the thesis entitled "NUMERICAL MODELLING OF OXYGEN AND CARBON DIOXIDE TRANSPORT IN THE PULMONARY CIRCULATION", being submitted by AZIM AMINATAEI for the award of the degree of 'Doctor of Philosophy' is a record of the original bonafide research work carried out by him. He has worked under our joint guidance and supervision and has fulfilled the requirements for the submission of this thesis. The results presented in this thesis have not been submitted in part or full to any other University or Institute for award of any degree/ diploma.

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ABSTRACT

The studies on the transport of oxygen and carbon dioxide leading to the oxygenation of blood in the pulmonary circulation are of physiological and pathological interests. They can also be helpful in improving the diagnostic procedures and for more reliable design of artificial oxygenators. The function of the lungs is to oxygenate the blood and to remove CO_2 from it. The transport of O_2 and CO_2 between the alveolar air and the blood flowing through the lung capillaries depend mainly on molecular diffusion, convection and the facilitated diffusion. Since the rate of gas exchange in pulmonary circulation is very fast, it is difficult to determine experimentally the effect of various physiological parameters on it. Therefore, the development of mathematical models is necessary for understanding the process of gas exchange in the lungs.

A mathematical model is proposed to deal with the simultaneous transport of O_2 and CO_2 in the pulmonary circulation. The model takes into account the transport of the species due to molecular diffusion, convection and the facilitated diffusion due to haemoglobin as a carrier of gases. The resulting system of governing equations together with the physiologically relevant boundary and entrance conditions is solved numerically by the four-point semi-implicit scheme. The rate at which blood gets oxygenated in the pulmonary capillaries is computed. It is found that, in the immediate neighbourhood of the entry, the facilitated diffusion is dominant over the molecular

diffusion. It is shown that the equilibration length increases in high altitude and muscular exercise. The resistance offered by the pulmonary membrane on the transport of gases in lungs, has been incorporated in the model. It is shown that the equilibration length increases as the resistance offered by the membrane increases.

The process of gas exchange in the pulmonary circulation has also been examined by incorporating the contribution of axial diffusion. The mathematical formulation leads to a coupled system of nonlinear elliptic partial differential equations (PDEs). A numerical scheme is described to solve such a system. It is found that the axial diffusion does not have an appreciable effect on the transport of the species in the blood. These studies were based on the steady-state condition. However, the transport of the gases by the blood is a function of time and therefore, the study has been extended to the nonsteady state. The formulation leads to a time-dependent coupled system of parabolic PDEs. A numerical scheme is described to solve the time-dependent nonlinear convective-diffusion equations. It is found that about .275 seconds are required for the transient recovery from the polluted to normal atmosphere where the venous O_2 partial pressure is increased suddenly from 31.2 mm Hg to 40 mm Hg. The time-dependent study has been applied to many situations.

In the above-mentioned studies, the blood has been treated as a homogeneous layer of haemoglobin solution. Due to the comparable size of the capillary and the erythrocyte (RBC), the blood can no longer be considered as a homogeneous fluid and hence, it is necessary to consider the blood flow as a two-phase flow consisting of cells and plasma. In order to analyze the effect of the cell free plasma layer near the wall on the oxygenation of blood in pulmonary capillaries, a two layer model has been considered. The coupled system of convective-diffusion equations together with the physiologically relevant boundary,entrance and interface conditions is solved numerically. It is shown that the thickness of the cell depleted layer affects the oxygenation process significantly.

These studies describing the uptake of O_2 by haemoglobin are based on the first order one step kinetics. However, the first order one step kinetics leads to a hyperbolic curve rather than the experimentally observed sigmoidial oxygen dissociation curve (ODC). Here, the nth order one step kinetics of O_2 uptake by haemoglobin has been proposed which leads to the sigmoidial ODC. The separate transport of CO_2 in the form of HCO_3^- in the plasma and RBC's is considered. The effect of PO_2, PCO_2 and plasma pH while modelling the simultaneous transport of O_2 and CO_2 in the pulmonary and the systemic circulations is accounted for. Further, the mathematical formulae for the ODC and CO_2 dissociation curve (CDC) have been developed. The ODC and CDC are consistent with the well-known Bohr and Haldane effects. The study is extrapolated to high altitude and hyperbaric conditions.

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