TWO-DIMENSIONAL MODELING AND ANALYSIS OF THIN ELECTROELASTIC STRUCTURES WITH APPLICATION TO ACTUATOR AND SENSOR DEVICE DESIGN

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Certificate

This is to certify that the thesis entitled, **'Two-dimensional Modeling and Analysis of Thin Electroelastic Structures with Application to Actuator and Sensor Device Design'** submitted by **Mr. Yadwinder Singh Joshan** to the **Indian Institute of Technology Delhi** for the award of degree of **Doctor of Philosophy** embodies original research work done by him under my supervision. In my opinion, the thesis work meets the requisite standards and the candidate is worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the institute. The contents of this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Place: New Delhi, India

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Yadwinder Singh Joshan

Abstract

In this work, two-dimensional asymptotic theories are presented for modeling and analysis of electroelastic structures in different operational regimes with application to design of actuator and sensor devices. Three types of electroelastic materials, namely, piezoelectric materials, flexoelectric materials and dielectric elastomers are analyzed for different transducer applications. The two-dimensional models are developed specifically for each material based on the type of electromechanical coupling and requirements of the application.

Piezoelectric materials are explored for the design of torsional and transverse shear sensors using a novel non-polynomial electromechanical shell theory. The 2D shell theory is utilized to analyze piezoelectric shear sensors made of low symmetry materials with simple patch geometries in different operating modes. The electric potential and displacement fields are assumed to be inverse-hyperbolic functions of the thickness coordinate. A new computationally efficient C^0 continuous deep shell finite element framework is developed starting from the principle of minimum potential energy and additional continuity requirements are imposed through the use of penalty parameter approach. The results are verified by comparing with 3D FEM results. It is also observed that the inverse-hyperbolic theory has higher accuracy compared to the third-order polynomial theory for same number of degrees of freedom. Parametric studies are subsequently performed to analyze the effect of geometric parameters and loading conditions on the sensor response of piezoelectric shell structures.

Flexoelectric materials are analyzed using a novel gradient electromechanical theory that incorporates both direct and converse flexoelectric effects. The two-way coupled

electromechanical theory is developed starting from a 3D variational formulation by considering an electric field-strain based free energy function. The formulation incorporates mechanical as well as electrical size effects. This theory is used to analyze flexoelectric beams and plates in actuator and sensor modes. A novel C^2 continuous finite element framework is developed to solve the beam and plate governing equations. Our finite element results are verified with analytical solutions for simply-supported boundary conditions. The computational framework is subsequently used to perform various parametric studies to analyze the effect of electrical and mechanical length scale parameters and geometric parameters on the response of the flexoelectric structures. Our simulation results also agree well with the trends observed in recent experiments.

Finally, dielectric elastomers are analyzed for the design of soft membrane actuators. The membrane actuators are modelled using an $O(h)$ non-linear membrane theory incorporating finite deformations. Specifically, the deformations in circular and cylindrical membrane actuators are analyzed under applied pressure and electric field. The limit point instability in electroelastic membranes due to applied loads is also investigated as these instabilities can lead to failure in dielectric elastomer actuators due to dielectric breakdown.

In summary, two-dimensional frameworks applicable for the computational design and analysis of dielectric electroelastic materials based actuators and sensors are developed in this work. The utility of these models is demonstrated by applying it to different devices based on the regime of operation.

सारांश

इस कार्य में वैद्युत प्रत्यास्थित ढांचे के प्रतिरूपन और विश्लेषण के विभिन्न संचालन व्यवस्था के अन्तर्गत प्रवर्तक और संवेदक उपकरणों के उपयोग के लिए द्वि-आयामी स्पर्शोन्मुख सिद्धांतो को प्रस्तुत किया गया है। दाब विद्युत प्रदार्थ, फ्लेक्सोइलेक्ट्रिक प्रदार्थ और परावैद्युत प्रत्यास्थित नामक तीन प्रकार के वैद्युत प्रत्यास्थित प्रदार्थों का विश्लेषण विभिन्न ट्रांसड्यूसर उपयोग के लिए किया गया है। विशेष रूप से विद्युतयांत्रिकी युग्मन और उपयोग के रूपरेखा पर आधारित प्रत्येक प्रदार्थ के लिए द्वि-आयामी नमूने विकसित किए गए है।

आधूर्ण और अनुप्रस्थ अपरूपन संवेदक की अभिकल्पना एक नई गैर बहुपदीय विद्युत यांत्रिकी शैल सिद्धांत द्वारा की गई है जिसमे दाब विद्युत पदार्थों का अन्वेषण किया गया है। द्वि-आयामी शैल सिद्धांत का उपयोग लघु समरूपक पदार्थ से बने साधारण पैच ज्यामिति वाले दाब विद्युत अपरूपन संवेदक के विश्लेषण में विभिन्न परिचालन तरीके मे किया गया है। विद्युत विभव और विस्थापन क्षेत्र को मोटाई निर्देशांक का विपरीत अतिपर्वलिक फलन माना गया है। न्यूतम स्तिथिज ऊर्जा के सिद्धांत से शुरुआत करके एक नई कुशल गणनात्मक सी^{0} सतत दीप शैल परिमित घटकीय रूपरेखा विकसित की गई है जिसमे पेनल्टी पैरामीटर का इस्तेमाल करते हुए अतिरिक्त सतत आवश्यकताएं अधिरोपित की गई है। परिणामों का सत्यापन त्रि-आयामी परिमित घटक तरीको के परिणामों से तुलना करके किया गया है। यह भी देखा जाता है कि समान संख्या की स्वतंत्र कोटी के लिए विपरीत अतिपर्वलक सिद्धांत त्रिकोटिए बहुपद सिद्धांत की तुलना में अधिक सटीकता रखता है। ज्यामितीय मापदंड और लदन अवस्थाओं का दाब विद्युत शैल ढांचे के संवेदकीय जवाब के प्रभाव का मापदंडिये विश्लेषण का अध्ययन किया गया है।

फ्लेक्सोइलेक्ट्रिक पदार्थों का विश्लेषण एक नवीनतम ग्रेडिएंट इलेक्ट्रो-मैकेनिकल सिद्धांत से किया गया है जिसमे दोनो अपरोक्ष और परोक्ष फ्लेक्सोइलेक्ट्रिक प्रभाव समलित है। एक विधुतिए क्षेत्र विकृति अधारित मुक्त ऊर्जा फलन को मान कर एक त्रि-आयामी परिवर्तनशील सूत्रीकरण से शुरुआत करते हुए दो तरफी जुड़े इलेक्ट्रो-मैकेनिकल सिद्धांत को विकसित किया गया है। यह सूत्रीकरण यांत्रिकी के साथ ही विद्युत आकार के प्रभाव शामिल करता है। यह सिद्धांत फ्लेक्सोइलेक्ट्रिक बीम और प्लेट का प्रवर्तक और संवेदक तरीके में विश्लेषण करती है।

बीम और प्लेट की नियामक समीकरणों को हल करने के लिए एक नवीनतम सी^{2} सतत परिमित घटकिए रूपरेखा विकसित की गई है। सरल समर्थित सीमा शर्तों के लिए हमारे परिमित घटकिए परिणाम को विश्लेषणात्मक हल से सत्यापित किया गया है। तत्पश्चात विद्युत और यांत्रिकीय लाम्बिक मापदंड पैमानों और ज्यामिति मापदंडों के प्रभाव का अध्ययन फ्लेक्सोइलेक्ट्रिक ढांचे के जवाब पर विभिन्न मापदंडों के अंतर्ग्रत गणनात्मक रूपरेखा का प्रयोग कार्ट3 हुए किया गया है। हमारे अनुकूलन परिणाम भी हाल के प्रयोग में देखे गए प्रवृत्तियों से अच्छी तरह से सहमत होते हैं।

अंतिम रूप में नरम परत प्रवर्तकों कि रचना के लिए परावैद्युत प्रत्यास्थलक का विश्लेषण किया गया है। एक औ(एच) गैर रेखीय परत सिद्धांत में परिमित विरूपण लेते हुए परत प्रवर्तक का प्रारूप किया गया है।विशेष रूप से गोलाकार और बेलनकार परत प्रवर्तकों की विकृतियों का विश्लेषण प्रयुक्त दाब और विद्युत क्षेत्र के तहत किया गया है। प्रयुक्त बल के कारण विद्युत प्रत्यास्थत परतों की सीमा स्थिरता को

भी जांचा गया है क्योंकि परावैद्युत विकार के कारण यह अस्थिरता परावैद्युत प्रत्यास्थलक प्रवर्तक की असफलता के लिए ज़िम्मेदार हो सकती है।

सारांश में परावैद्युत विद्युत प्रत्यास्थत पदार्थ आधारित प्रवर्तक और संवेदक के संगणनीय रचना और विश्लेषण के लिए उपयोगी द्वि-आयामी ढांचे इस काम में विकसित किए गय है। इन मॉडलों की उपयोगिया संचालन व्यवस्था पर आधारित विभिन्न उपकरणों पर लागू करके दिखाई गई है।

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$\Delta\phi$ Electrostatic potential applied across the electroelastic layer, see equation (4.167), page 124

$\alpha_1, \alpha_2, \alpha_3$ Curvilinear coordinates, see equation (3.6), page 24

δ Dimensionless material parameter, see equation (5.43), page 148

δ_{il} Kronecker delta function, see equation (4.39), page 75

ϵ_0 Electrical permittivity in free space, see equation (5.9), page 139

ϵ_{ij} Components of dielectric permittivity matrix for piezoelectric material ($i, j = 1, 2, 3$), see equation (3.26), page 30

μ_1, μ_2 Penalty parameters, see equation (3.37), page 33

Ω Free energy function, see equation (4.1), page 67

ω Work done by the externally applied force, see equation (4.1), page 67

Φ Electrostatic potential across piezoelectric layer, see equation (3.22), page 29

ϕ Scalar electrostatic potential, see equation (4.3), page 68

ϕ_1, ϕ_2 Components of shear rotation, see equation (3.17), page 26

Π Potential energy of the coupled electromechanical system, see equation (3.29), page 31

ρ_s Surface charge density, see equation (3.31), page 31

ρ_v Applied volumetric charge density, see equation (4.8), page 69

S_I Strain components in Voigt notation ($I = 1, 2, 3, 4, 5, 6$), see equation (3.6), page 24

T_I	Stress components in Voigt notation ($I = 1, 2, 3, 4, 5, 6$), see equation (3.25), page 30
$\overline{\Phi}$	Nondimensional potential difference, see equation (3.71), page 49
$\overline{w}$	Nondimensional deflection, see equation (4.167), page 124
$\mathbf{T}$	Cauchy stress tensor, see equation (5.3), page 138
λ_j	Principle stretches (($j = 1, 2, 3$)), see equation (5.44), page 148
B_{ij}	Higher-order electric displacement tensor, see equation (4.6), page 68
c	Young's modulus, see equation (4.165), page 117
c_{ijkl}	Stiffness tensor, see equation (4.29), page 73
D_i	Components of electric displacement vector ($i, j = 1, 2, 3$), see equation (3.26), page 30
e_{ijk}	Piezoelectric coupling tensor, see equation (4.29), page 73
e_{iJ}	Components of piezoelectric coupling matrix, see equation (3.26), page 30
E_i	Components of electric field vector ($i, j = 1, 2, 3$), see equation (3.25), page 30
$f(\alpha_3)$	Non-polynomial shear strain function, see equation (3.17), page 26
f_i^b	Body force vector, see equation (4.8), page 69
f_{ijkl}	Direct flexoelectric coupling tensor, see equation (4.29), page 73
G	Shear modulus, see equation (5.43), page 148
g_{ijklmn}	Higher-order stiffness tensor, see equation (4.29), page 73
G_{ijk}	Strain gradient Tensor, see equation (4.1), page 67
h	Thickness of the structure, see equation (3.18), page 26
h_{ijkl}	Converse flexoelectric coupling tensor, see equation (4.29), page 73
H_{ijk}	Higher-order stress tensor, see equation (4.6), page 68
I_1, I_2, I_3	Invariants of Cauchy deformation tensor, see equation (5.43), page 148

J	Determinant of deformation gradient tensor, see equation (5.3), page 138
k_{ijkl}	Higher-order electric permittivity tensor, see equation (4.29), page 73
K_{ij}	Electric field gradient tensor, see equation (4.1), page 67
l_0^e, l_1^e	Electric length scale parameters, see equation (4.51), page 77
l_0, l_1, l_2	Mechanical length scale Parameters, see equation (4.47), page 76
q_3	Applied transverse mechanical load, see equation (3.31), page 31
q_{1s}, q_{2s}	Applied in-plane shear loads, see equation (3.31), page 31
Q_{IJ}	Components of stiffness matrix for piezoelectric materials ($I, J = 1, 2, 3, 4, 5, 6$), see equation (3.25), page 30
R_1, R_2	Principal radii of curvature of a doubly curved shell, see equation (3.6), page 24
S_{ij}	Infinitesimal strain tensor, see equation (4.1), page 67
t_i^a	Applied traction vector, see equation (4.8), page 69
T_{ij}	Cauchy stress tensor, see equation (4.6), page 68
u_0	Mid-plane displacement along α_1, see equation (3.17), page 26
u_i	Components of displacement vector ($i = 1, 2, 3$), see equation (3.6), page 24
U_T	Total internal energy, see equation (3.29), page 31
v	Poisson's ratio, see equation (4.165), page 117
v_0	Mid-plane displacement along α_2, see equation (3.17), page 26
W	External work done, see equation (3.29), page 31
w_0	Mid-plane displacement along α_3, see equation (3.17), page 26
$\overline{\mathbf{d}}$	Global field variable vector, see equation (3.60), page 40
$\overline{\mathbf{F}}_\mathbf{g}$	Global force vector, see equation (3.64), page 41
$\overline{\mathbf{K}}_\mathbf{g}$	Global stiffness matrix, see equation (3.60), page 40

$\mathbf{C}$	Right cauchy green stress tensor, see equation (5.42), page 148
$\mathbf{R}$	Orthogonal rotational tensor, see equation (5.46), page 149
$\mathbf{U}$	Right stretch tensor, see equation (5.46), page 149
$\mathbf{V}$	Left stretch tensor, see equation (5.46), page 149
$\mathbf{X}$	Position of a point in reference configuration, see equation (5.11), page 140
$\mathbf{x}$	Position of a point in current configuration, see equation (5.39), page 147
$\mathbf{e}_f$	Electric field in free space, see equation (5.9), page 139
$\mathbf{F}$	Deformation gradient tensor, see equation (5.3), page 138
$\mathbf{I}$	Identity Tensor, see equation (5.9), page 139
$\mathbf{P}$	Piola stress tensor, see equation (5.1), page 137
$\mathbf{T}_m$	Maxwell stress tensor, see equation (5.9), page 139
$\mathbf{W}$	Free energy function, see equation (5.43), page 148

Acronyms

CBC	Charge Boundary Conditions
CBT	Classical Beam Theory
CLPT	Classical Laminated Plate Theory
CLST	Classical Laminated Shell Theory
DKT	Discrete Kirchhoff Technique
EAP	Electroactive Polymers
FE	Finite Element
FEM	Finite Element Method
FSDT	First Order Shear Deformation Theory
HPBC	Higher-order Potential Boundary Conditions
HSDT	Higher-order Shear Deformation Theory
HSL	Hydrostatic Load
HSM	Hybrid Stress Method
IHSDT	Inverse Hyperbolic Shear Deformation Theory
PBC	Potential Boundary Conditions
PFRC	Piezoelectric Fiber Reinforced Composite
ODE	Ordinary Differential Equation
SSL	Sinusoidal Load
UDL	Uniformly Distributed Load