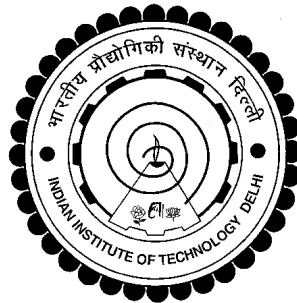


**AUGMENTATION OF
PARTIALLY-AVERAGED NAVIER STOKES
METHOD USING NON-LINEAR EDDY
VISCOSITY CLOSURE**

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**DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
JULY 2020**

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by

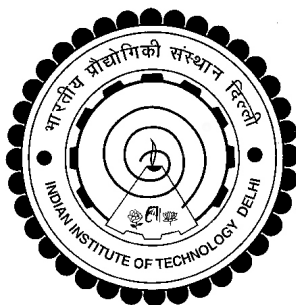
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Submitted

in fulfillment of the requirements for the degree of Doctor of Philosophy

to the



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CERTIFICATE

This is to certify that the thesis entitled “**Augmentation of Partially-Averaged Navier Stokes Method using Non-Linear Eddy Viscosity Closure**”, being submitted by **Sagar Saroha** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy**, is a record of the bonafide research carried out by him which has been prepared under our supervisions in conformity with the rules and regulations of the Indian Institute of Technology Delhi. The research reports and results presented in the thesis has not been submitted for any degree or diploma in any other university or institute.



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ABSTRACT

In recent years partially-averaged Navier-Stokes (PANS) method has emerged as a viable bridging method of turbulence. PANS is a filtering based, scale-resolving bridging method. The PANS method has been successfully evaluated in a variety of both canonical and practical flows of interest. Conventional PANS method employs linear eddy viscosity based closure for unclosed turbulent stress terms. PANS based on linear closure (linear PANS) overcomes the scale-resolving inadequacies of the Reynolds-averaging, but still suffers from the limitations arising from linear constitutive modeling of turbulent stresses. In linear models, unclosed turbulent stresses are modeled as a linear function of the mean strain-rate tensor. Experimental and computational works report that linear constitutive modeling is insufficient to capture essential flow physics in a variety of turbulent separated flows. In non-linear eddy viscosity models, unclosed turbulent stresses are related to the higher-order terms involving mean strain-rate and mean rotation-rate tensors as well, besides the linear terms. The non-linear models are reported to perform better than the linear models in case of Reynolds-averaged Navier Stokes (RANS) simulations of many complex turbulent flows.

In our attempt to enhance the performance of the PANS model for separated turbulent flows, the specific tasks taken up in this thesis can be listed as:

1. Extraction of non-linear eddy viscosity based PANS closure.
2. Sensitization of thermal flux terms using non-linear stresses.
3. Ordinary differential equation (ODE) based analysis of the non-linear PANS model in the absence of the transport terms.

4. Implementation of PANS methodology in computational fluid dynamics (CFD) solver packages like ANSYS FLUENT[®] and OpenFOAM[®].
5. Comparative evaluation of the linear PANS vs. non-linear PANS in canonical flow test cases.

Among various higher-order models available, in this work, we choose the non-linear model proposed by [Shih *et al.* \(1993\)](#). Shih’s non-linear model uses the standard k - ϵ RANS model and a non-linear algebraic equation for unclosed Reynolds stress tensor. We have extracted non-linear PANS based on Shih’s non-linear closure using the definition of the filter parameters and underlying PANS assumptions. Besides modeling the turbulent stresses, we have proposed a novel non-linear sensitization of the thermal flux to account for the effect of non-linearity in temperature equation as well.

An ODE-based analysis of the extracted non-linear PANS model is performed to examine the production-dissipation dynamics in the absence of transport processes. This analysis involved an extensive examination of the influence of PANS filter control parameter and type of resolved velocity gradient tensor on: (i) the magnitude of the turbulent stress; (ii) anisotropy of the turbulent stress tensor, and (iii) relative orientation of turbulent stress tensor and the resolved strain-rate tensor.

Further, validation and assessment of the non-linear PANS methodology developed in this research are performed using open-source CFD tool OpenFOAM[®]. Code development for OpenFOAM involved the inclusion of near-wall resolving treatment, changes in the solver to include energy equation and re-writing of existing RANS models’ library to accommodate both linear and non-linear PANS. Implementation of linear PANS methodology in commercial CFD solver ANSYS FLUENT[®], required for one sub-study, is done with the help of available RANS model structure.

Performance of non-linear PANS is subjected to evaluation in two canonical flow fields — flow past a square cylinder and flow past a sphere. Assessment is

performed by comparing the performance of linear PANS vs. non-linear PANS. The influence of the reduction in the value of the filter control parameter is also examined. Evaluations are performed in terms of both hydrodynamics quantities (like drag coefficient, pressure coefficient, skin friction coefficient, velocity profiles, vortex structures, wake contours) and heat transfer aspects (like Nusselt number variation, temperature profiles). Comparison is made against the available experimental, direct numerical simulation (DNS) and large eddy simulation (LES) results. The results of these comparisons highlight the enhancement in the performance of the PANS due to non-linear closure over the conventional linear based closure in the considered test-cases of separated flows.

Bridging methods are computationally cost-efficient, and improvement in their predictive accuracy will widen their applicability for other variety of complex flows. The author believes that this study will motivate more users to further explore and evaluate the integration of bridging methods like PANS with other non-linear and even explicit algebraic Reynolds stress models (EARSMS) as well.

सारांश

हाल के वर्षों में पार्शियली एवरेजड नॉविएर स्टोक्स (PANS) मेथड व्यावहारिक वैकल्पिक ब्रिजिंग विधि के रूप में उभर कर आया है। PANS फ़िल्टरिंग-आधारित स्केल रेसॉल्विंग ब्रिजिंग विधि है। PANS को विहित और व्यावहारिक दोनों प्रकार के प्रवाह में सफलतापूर्वक मूल्यांकित किया गया है। पारंपरिक PANS मेथड लीनियर एडी विस्कोसिटी पर आधारित असुलझे टर्बुलेन्ट स्ट्रैस टर्म्स की मॉडलिंग करता है। लीनियर मेथड पर आधारित PANS (लीनियर PANS) रेनोल्ड्स-एवरेजिंग की अप्रयाप्तियों को पूरा करता है, लेकिन वह लीनियर विधान से टर्बुलेन्ट स्ट्रैस की मॉडलिंग करने पर उत्पन्न हुई अक्षमताओं को काबू नहीं कर पाता। इन लीनियर मॉडल्स में असुलझे टर्बुलेन्ट स्ट्रैसेस को स्ट्रेन-रेट टेन्सर के लीनियर फंक्शन के तौर पर मॉडल किया जाता है। प्रयोगात्मक और कम्प्यूटेशनल काम स्थापित करते हैं की अशांत अलग प्रवाह की विविधता में आवश्यक प्रवाह भौतिकी का अनुमान लगाने के लिए लीनियर विधान द्वारा मॉडलिंग नाकाफी है। नॉन-लीनियर एडी विस्कोसिटी मॉडल्स असुलझे टर्बुलेन्ट स्ट्रैस टर्म्स को औसत स्ट्रेन-रेट और औसत रोटेशन-रेट टेंसर्स की लीनियर टर्म्स के अलावा उच्च कक्ष की टर्म्स के तौर पर सम्बंधित करता है। रेनोल्ड्स एवरेजड नॉविएर स्टोक्स (RANS) सिमुलेशनस के जटिल अशांत प्रवाह के मामले में नॉन-लीनियर मॉडल्स को लीनियर मॉडल्स से बेहतर प्रदर्शन करता पाया गया है।

इस प्रयास में अशांत अलग प्रवाह के लिए PANS मॉडल के प्रदर्शन में सुधार करने के लिए निम्नलिखित कार्य अपनाए गए हैं:

1. नॉन-लीनियर एडी विस्कोसिटी पर आधारित PANS मॉडल का निष्कर्षण
2. नॉन-लीनियर स्ट्रेस द्वारा थर्मल प्रवाह का संवेदीकरण
3. साधारण अंतर समीकरण (ODE) पर आधारित नॉन-लीनियर PANS मॉडल का परिवहन टर्म्स की अनुपस्थिति में आंकलन
4. PANS प्रक्रिया का अभिकलनात्मक जटिलता द्रव गतिकी के सॉल्वर पैकेज ANSYS FLUENT और OpenFOAM में कार्यान्वयन
5. लीनियर PANS और नॉन-लीनियर PANS का विहित प्रकार के प्रवाह में तुलनात्मक मूल्यांकन

विभिन्न उच्च कक्ष मॉडल्स में से इस काम में Shih (१९९३) के नॉन-लीनियर मॉडल का चुनाव किया गया है। Shih का नॉन-लीनियर मॉडल स्टैंडर्ड $k-\epsilon$ RANS विधि और नॉन-लीनियर बीजगणितीय समीकरण का प्रयोग किया गया है असुलझे टर्बुलेन्ट स्ट्रैस टर्म्स के लिए। Shih के नॉन-लीनियर मॉडल के प्रयोग से फ़िल्टर मापदंडों की परिभाषा और निहित पूर्वधारणायें के अनुसार नॉन-लीनियर PANS का निष्कर्षण किया गया है। टर्बुलेन्ट स्ट्रैस टर्म्स की मॉडलिंग के अलावा, तापमान के समीकरण में थर्मल प्रवाह के लिए एक नए नॉन-लीनियर संवेदीकरण प्रस्तावित किया गया है।

आगे इस अनुसंधान में विकसित नॉन-लीनियर PANS विधि का सत्यापन और मूल्यांकन खुले स्रोत अभिकलनात्मक जटिलता द्रव गतिकी (CFD) के साधन OpenFOAM में किया गया है। OpenFOAM कोड में लीनियर तथा नॉन-लीनियर PANS को समायोजित करने के विकास के लिए दीवार के निकट को सुलझाने का समावेश, तापमान समीकरण में सॉल्वर में बदलाव और वर्तमान में उपस्थित RANS सॉल्वर के लाइब्रेरी का पुनर्लेखन किया गया है। एक उप अध्ययन हेतु व्यावसायिक CFD सॉल्वर ANSYS FLUENT में लीनियर PANS का कार्यान्वयन उपलब्ध RANS मॉडल संरचना की सहायता से किया गया है।

नॉन-लीनियर PANS का प्रदर्शन दो प्रवाह क्षेत्र में मूल्यांकित किया गया है - एक वर्ग सिलेंडर के पिछले प्रवाह और एक वर्ग गैड के पिछले प्रवाह में। लीनियर PANS और नॉन-लीनियर PANS में तुलनात्मक आकलन किया गया

है। फ़िल्टर नियंत्रण पैरामीटर के मूल्य में कमी का प्रभाव की भी जांच की गई है। दोनों मात्रा के संदर्भ में मूल्यांकन किया जाता है हाइड्रोडायनामिक्स (जैसे खींचें गुणांक, दबाव गुणांक, त्वचा घर्षण गुणांक, वेग प्रोफाइल, भंवर संरचनाओं, जगा आकृति) और गर्मी का हस्तांतरण (जैसे Nusselt संख्या में बदलाव, तापमान प्रोफाइल)। पूर्व के उपलब्ध परिणाम के खिलाफ तुलना की गई हैं - क्रमिक, प्रत्यक्ष संख्यात्मक सिमुलेशन (DNS) और बड़े एडी सिमुलेशन (LES)। इन परिणामों अलग प्रवाह के परीक्षण-मामलों में लीनियर विधान की तुलना में नॉन-लीनियर विधान से PANS के प्रदर्शन में हुई वृद्धि पर प्रकाश डाला गया है।

ब्रिजिंग विधियां कम्प्यूटेशनल रूप से लागत-कुशल हैं, और उनकी भविष्यवाणी की सटीकता में हुए सुधार एवं अन्य प्रकार के जटिल प्रवाह के लिए उन की उपयुक्तता को बढ़ाएंगी। लेखक का मानना है कि यह अध्ययन अधिक उपयोगकर्ताओं को प्रेरित करेगा अन्य गैर रेखीय विधियों के साथ PANS जैसे ब्रिजिंग विधियों के एकीकरण और यहां तक कि स्पष्ट बीजीय रेनॉल्ड्स तनाव मॉडल (EARSIM) के भी मूल्यांकन में।

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Nomenclature

Notations

$\dot{q}$	heat flux
$\langle \rangle$	partial-averaging filter
$\langle \rangle_{mean}$	time averaged mean quantity
$\langle p \rangle$	filtered pressure
$\langle S_{ij} \rangle$	filtered strain-rate tensor
$\langle T \rangle$	filtered temperature
$\langle V_i \rangle$	filtered velocity
ρ	density
C_D	coefficient of drag
C_f	coefficient of skin friction
C_p	coefficient of pressure
f_e	ratio of the unresolved dissipation rate of turbulent kinetic energy (PANS) to rate of dissipation of turbulent kinetic energy (RANS)
f_k	ratio of the unresolved turbulent kinetic energy (PANS) to turbulent kinetic energy (RANS)
F_x	force along x-direction
K	thermal heat conductivity
k	turbulent kinetic energy
k_u	unresolved turbulent kinetic energy
n	wall normal distance
Nu	Nusselt number
p	instantaneous pressure

p_0	free-stream pressure
p_∞	free-stream pressure
P_u	unresolved production term of turbulent kinetic energy
$p_{surface}$	pressure value on surface
Pr	Prandtl number of the fluid
Pr_T	turbulent Prandtl number
S_{ij}	strain-rate tensor
T	instantaneous temperature
t	time
T_0	free-stream temperature
U_∞	free-stream velocity
U_x	instantaneous velocity along x -direction
V_∞	free-stream velocity
V_i	instantaneous velocity vector
x	spatial coordinates
Re	Reynolds number

Greek Symbols

ϵ	rate of dissipation of turbulent kinetic energy
ϵ_u	unresolved rate of dissipation of turbulent kinetic energy
η_r	viscosity ratio
Γ	stress ratio (SR)
$\langle \Omega_{ij} \rangle$	filtered rotation-rate tensor
μ	dynamic viscosity
ν	kinematic viscosity
ν_t	turbulent eddy viscosity
ν_u	unresolved eddy viscosity
ω	specific rate of dissipation of the turbulent kinetic energy
Ω_{ij}	rotation-rate tensor
$\tau^{anisotropic}$	anisotropic part of turbulent stress

$\tau^{viscous}$ filtered viscous stress

Abbreviation

AC	axisymmetric contraction
AE	axisymmetric expansion
AR	alignment ratio
BFS	backward facing step
CFD	computational fluid dynamics
DES	detached eddy simulation
DNS	direct numerical simulation
EARSM	explicit algebraic Reynolds stress model
HS	homogeneous shear
LES	large eddy simulation
LEVSM	linear eddy viscosity model
NLEVSM	non linear eddy viscosity model
ODE	ordinary differential equation
PANS	partially averaged Navier Stokes
PISO	pressure implicit with splitting of operator
PS	plain strain
RANS	Reynolds-averaged Navier Stokes
SIMPLE	semi-implicit method for pressure-linked equations
SR	stress ratio, Γ