

APPROXIMATIONS OF RECURSIVE
DIGITAL FILTERS

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ABSTRACT

The approximations of ideal magnitude and phase characteristics suitable for recursive digital filtering have been considered. A critical comparison has been made of several existing approximations and some new and improved ones have been presented.

The frequency characteristics of low-pass sine and tan filters, in the transition ^{region} and stop-band, are examined and compared with that of the generating analog filters. The parameters used for comparison are cutoff slope, asymptotic attenuation and transition ratio. Several methods of designing band-pass tan filters and a synthesis procedure of band-pass sine and cosine filters from the squared magnitude specification have been presented. A new band-pass magnitude function is obtained by combining the sine and cosine filters with better characteristics than the tan filter under certain conditions.

Transition and stop-band behaviour of sine and tan filters with multiple pairs of coincident transmission zeros in the stop-band have been investigated next. Filters with both maximally flat and equiripple pass-bands have been considered. It has been shown that the introduction of transmission zeros at an appropriate frequency

(ii)

improves the cutoff slope of a maximally flat filter considerably over that of the Butterworth filter. It is also demonstrated that the Butterworth tan filter is a special case of maximally flat sine filter with transmission zeros at half the sampling frequency of multiplicity equal to the order of the filter. In the equiripple case, the results obtained are similar to those of maximally flat filters.

Finally a direct synthesis procedure of recursive filters with nearly linear phase characteristic has been given. The phase characteristic is shown to be linear with a maximum periodic error, which can be kept as small as desired. The stop-band behaviour of these filters has also been examined and an upper bound obtained for the asymptotic db-loss.

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	i
LIST OF IMPORTANT SYMBOLS •	vii
<u>CHAPTER 1</u> BASIC PRINCIPLES OF DIGITAL FILTERS AND SCOPE OF THE THESIS	
1.1 Introduction	1
1.2 Principles of Digital Filters	3
1.3 Transfer Function and Realization of Digital Filters	5
1.4 Recursive and Nonrecursive Filters	7
1.5 Synthesis of Recursive Filters	8
1.5.1 Impulse invariant technique	8
1.5.2 Bilinear transformation	9
1.5.3 Design of digital filters from squared magnitude specifications	10
1.6 Synthesis of Nonrecursive filters	11
1.7 Scope of the Thesis	12
<u>CHAPTER 2</u> A COMPARISON OF LOW-PASS TAN AND SINE FILTERS WITH THE GENERATING ANALOG FILTER	
2.1 Introduction	15
2.2 Cutoff Slope	16
2.3 Asymptotic Behavior	18
2.4 Transition Ratio	21

	<u>Page</u>
2.5 Transfer Function	23
2.6 Conclusion	24
<u>CHAPTER 3</u> SYNTHESIS OF BAND-PASS FILTERS	
3.1 Introduction	25
3.2 Synthesis of Band-Pass Tan Filters	25
3.3 Band-Pass Sine Filters	28
3.4 Band-Pass Cosine Filters	31
3.5 Sine-Cosine Filter with Band-Pass Response	32
3.6 Conclusion	34
<u>CHAPTER 4</u> MAXIMALLY FLAT LOW-PASS FILTERS WITH IMPROVED CUPOFF CHARACTERISTICS	
4.1 Introduction	36
4.2 Maximally Flat Low-Pass Filters	37
4.2.1 Filter magnitude function	37
4.2.2 Cutoff slope	38
4.2.3 Attenuation in stop-band	40
4.3 Maximally Flat Sine Filter with Trans- mission Zeros at $\omega_s / 2$	42
4.4 Realization Consideration	45
4.5 Conclusion	48
Appendix 4.1	49
Appendix 4.2	51

<u>CHAPTER 5</u>	SHARP CUTOFF LOW-PASS FILTERS WITH EQUIRIPPLE PASS-BAND	
5.1	Introduction	53
5.2	Low-Pass Filters with Equiripple Pass-Band	53
5.2.1	Filter magnitude function	53
5.2.2	Cutoff slope	57
5.3	Equiripple Filters with Number of Poles Equal to the Number of Zeros	60
5.3.1	Filter function	60
5.3.2	Stop-band attenuation	60
5.4	Conclusion	65
	Appendix 5.1	66
	Appendix 5.2	67
<u>CHAPTER 6</u>	SYNTHESIS OF LINEAR PHASE RECURSIVE FILTERS	
6.1	Introduction	70
6.2	Linear Phase Low-Pass Characteristic	72
6.3	Transfer Function and Delay	74
6.4	Stop-Band Attenuation	76
6.5	Examples	78
6.6	Conclusion	79
	Appendix 6.1	80
	Appendix 6.2	80
<u>CHAPTER 7</u>	CONCLUSIONS	
7.1	Main Results of the Thesis	82

7.2 Concluding Remarks and Scope for

Further Work

86

REFERENCE S

89