

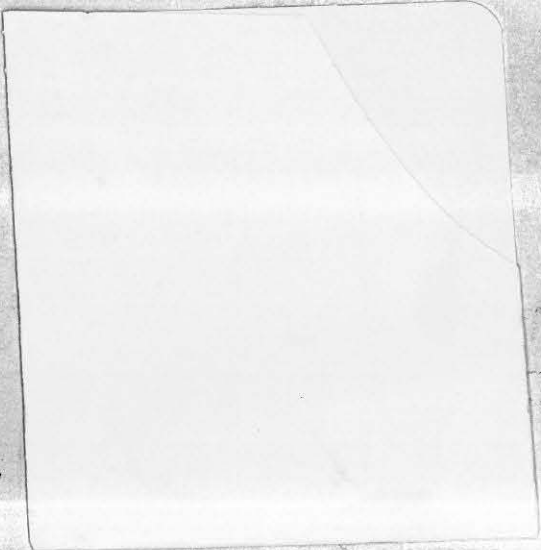
BOUNDARY LAYER FLOWS AND HEAT TRANSFER VIA QUASILINEARIZATION

PRAMOD KUMAR



**DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY, DELHI**

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BY
PRAMOD KUMAR

THESIS SUBMITTED TO
THE INDIAN INSTITUTE OF TECHNOLOGY, DELHI
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DOCTOR OF PHILOSOPHY



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INDIAN INSTITUTE OF TECHNOLOGY, DELHI**

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(PRAMOD KUMAR)

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ABSTRACT

In this work the Quasilinearization technique is utilized to study the boundary layer flows and heat transfer in incompressible and compressible fluids. As this technique, along with the principle of superposition, is expected to be a very powerful tool for solving non-linear two point, more generally multi-point, boundary value problems (BVPs) we have chosen some of those problems which can be reduced to systems of two-point non-linear BVPs, namely

- I Flow and heat transfer in the boundary layer due to bodies of revolution,
- II Axis-symmetric hydromagnetic stagnation point flow and heat transfer,
- III Compressible laminar boundary layer over a yawed infinite cylinder with heat transfer and arbitrary Prandtl number,
- IV Natural convection from a vertical hot plate embedded in a saturated porous media,
- V Radiative magnetohydrodynamic channel flow with thermal conductivity,
- VI Heat transfer effects to the stagnation point from a dissociating binary mixture in frozen flow.

By adopting a power-series solution in the transformed coordinates, the general three dimensional boundary layer equations governing the flow corresponding to Prob. I, are reduced to systems of non-linear and linear boundary value problems. While the similarity transformation is utilized to reduce the governing equations of Prob. II to non-linear BVPs, Illingworth-Stewartson transformation, in addition to certain other assumptions, is employed to reduce the corresponding equations of Prob. III to non-linear BVPs. For Prob. IV a similarity transformation and a power-series solution were necessary to get the desired form of BVPs. The governing equations of Prob. V were already in the desired form whereas for Prob. VI the cartesian coordinate system is transformed to the natural coordinate system so that the dependent variables become separable and ordinary differential equations result.

After having reduced the problems under consideration to their respective BVPs, the quasilinearization is used to linearize the non-linear systems. Subsequently, the fourth order Runge-Kutta integrator is used, in combination with superposition, to obtain the solutions of the systems of linear BVPs. As Probs. I - II give rise to the cases of failure of superposition principle, that necessitated the

remedial measures, other appropriate formulae are adopted to circumvent the difficulties. Superiority of the quasilinearization is noted over other shooting method in Prob. II. The results of Prob. III are highly encouraging and one is forced to repose great confidence in the reliability of quasilinearization technique.

The quantities of physical and engineering interest are determined in all the above mentioned problems. By choosing these problems our efforts are more than rewarded in establishing the efficacy and desirability of quasilinearization technique which completely eliminates the trial-and-error approach adopted so far in handling non-linear boundary value problems.

PART--A